## Measuring Option Liquidity<sup>\*</sup>

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#### Abstract

We compare all commonly employed transaction cost measures in options markets. Standard measures do not clearly identify the financial crisis, suggest that low-volume ITM options are much more liquid than high-volume ATM options, and are highly sensitive to small variations in option moneyness. We show that normalizing the relative bid-ask spread by the option elasticity accounts for the direct cost of implementing the replicating portfolio and ensures that the measure reflects the option's economic exposure. In our comparisons, this elasticity-adjusted spread outperforms alternative measures. It varies with financial market stress, is highly correlated with equity market liquidity, and shows high cross-sectional correlations with underlying liquidity, underlying market capitalization, and volatility. It is lowest for ATM and ITM options and much more robust to small variations in moneyness. A low-frequency and computationally cheap approximation of our measure performs well and better than the alternatives.

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Options markets play a critical role in global financial markets, providing essential tools for risk management, hedging, and speculation. Reflecting their increasing importance, equity option trading volumes in the U.S. surged dramatically, exceeding 10 billion contracts in 2023 – a 128% increase since 2019. This growth, driven by rising retail investor participation and institutional reliance on derivatives for comprehensive risk management and portfolio completion, underscores the importance of options markets. However, despite their growing economic significance, trading options remains costly compared to their underlying securities. This discrepancy arises primarily due to the embedded complexity and leverage associated with option positions. Traditional liquidity measures, such as the quoted bid-ask spread, have significant limitations in capturing these complexities, often resulting in misleading assessments of liquidity, particularly during periods of market stress or across varying degrees of moneyness.

Standard estimates of option transaction costs have three key issues: they fail to highlight liquidity stress periods like the financial crisis and have low correlations with economic fundamentals, they often suggest that low-volume ITM options are much more liquid than high-volume ATM options, and they are overly sensitive to small variations in moneyness. This paper addresses these problems with three contributions. First, we introduce a new liquidity measure that directly links transaction costs to the size of the replicating portfolio. The new measure is theoretically grounded and leads to a more robust empirical quantification of liquidity. Second, we compare existing liquidity measures, demonstrating the superiority of our approach in capturing economic fundamentals and market stress. Third, we develop computationally efficient approximations that make our measure practical for settings with daily data.

In equity markets, liquidity is commonly measured through the *bid-ask spread* that represents the difference between the highest price a buyer is willing to pay (*bid*) and the lowest price a seller is willing to accept (*ask*). This spread serves as a direct indicator of market liquidity: narrower spreads typically signify more liquid markets, while wider spreads suggest reduced liquidity and higher trading costs. To make liquidity measures comparable across different stocks and market conditions, the spread is often normalized by the stock price, resulting in the *relative bid-ask spread* (i.e., the spread divided by the mid-price). This normalization accounts for differences in stock prices, enabling better comparisons of liquidity across equities and varying market environments.

In options markets, using the relative spread to measure liquidity appears intuitive but requires additional considerations due to the complex nature and non-linearities of options. Unlike equities, where price serves as a sufficient proxy for position risk, options carry embedded leverage and are sensitive to multiple option-specific risk factors, such as delta, gamma, vega, and theta (time decay). Consequently, these characteristics make the bid-ask spread a potentially misleading indicator of liquidity, especially in relative comparisons. Furthermore, options can be replicated through costly dynamic hedging involving the underlying asset and risk-free bonds, which significantly differentiates them from equities.

The cost of implementing this replicating portfolio is directly tied to option liquidity. For instance, if the underlying stock has a wide bid-ask spread, the cost of adjusting the hedge will be higher, leading to increased transaction costs for market-makers. Consequently, the option's bid-ask spread should reflect these hedging costs. From this perspective, the quoted bid-ask spread is a good starting point for a liquidity measure. However, as in equities, normalizing the spread is essential to make liquidity comparable across options with different characteristics. The challenge lies in determining the appropriate normalization, given that option prices can be extremely small and exhibit a non-linear relationship with respect to underlying risk factors. Relatedly, the traditional relative bid-ask spread in options overlooks the option's embedded leverage – arising from the fact that a small change in the underlying can translate into an outsized percentage change in the option's price.

In this paper, we introduce the *Elasticity-Adjusted Spread (EAS)*, a liquidity measure that directly links trading costs to the stock-market exposure of the replicating portfolio. The EAS scales the relative bid-ask spread by the absolute value of the option's elasticity  $|\Delta| \frac{S}{O}$ , or, more simply, scales the dollar bid-ask spread by the *delta-weighted exposure* to the underlying asset  $|\Delta| \cdot S$ . By normalizing the bid-ask spread through the option's delta and the underlying price, the EAS directly ties transaction costs to the size of the replicating portfolio. This alignment ensures that the measure reflects the true economic exposure of the position (the share of the underlying needed to hedge), rather than being distorted by the sometimes tiny price of an option relative to the underlying asset. It is calculated as:

$$EAS = \frac{1}{|\Delta|\frac{S}{O}} \frac{\text{Bid-Ask Spread}}{O} = \frac{\text{Bid-Ask Spread}}{|\Delta| \cdot S}.$$
 (1)

The core idea of the normalization is best understood within a framework that traces option illiquidity to a single source: transaction costs incurred through hedging in the underlying asset. These transaction costs ultimately increase the replication cost of the option, which, to keep things simple, are assumed to be well approximated by the initial hedge costs, calculated as the underlying asset's relative transaction cost multiplied by  $|\Delta| \cdot S$ . In dollar terms, the replication costs – and thus the illiquidity of the option – ceteris paribus increase with moneyness. If we were to normalize by the option price, the normalized replication costs would be lowest for inthe-money options in a Black-Scholes framework, making ITM options relatively more liquid. This theoretical result almost never aligns with how illiquidity across moneyness is perceived in practice.

Alternatively, for the EAS, the normalized replication costs for a given underlying asset do not depend on the option's moneyness and directly correspond to the bid-ask spread of the underlying asset, i.e., the illiquidity of the underlying stock. This relationship serves as the benchmark in our options illiquidity measure. If the actual replication costs exceed this benchmark, our measure indicates greater illiquidity. Essentially, the EAS expresses options illiquidity in terms of the underlying stock's illiquidity. Within this framework, the EAS can be interpreted as the implicit bid-ask spread of the stock that explains the option's actual dollar bid-ask spread. By expressing option illiquidity on the same scale as stock illiquidity, the EAS is directly comparable to the traditional bid-ask spread of the underlying stock.<sup>1</sup> Therefore, to our knowledge, the EAS is the first liquidity measure that not only enables a meaningful liquidity comparison between options, but also allows to compare liquidity between option and underlying markets.

The drawbacks of standard liquidity measures applied to options markets have been documented before. In response, researchers have either adapted traditional measures or proposed entirely new ones. However, as of this paper, no comparative analysis of these different measures and approaches exists. For example, based on the dual relationship between option price and implied volatility, Hsieh and Jarrow (2019) and Chaudhury (2015) propose measuring the bid-ask spread in terms of implied volatility, either in absolute terms or normalized by the mid implied volatility or realized volatility. Additionally, Chaudhury (2015) suggests a spread relative to dollar volatility, and Grundy et al. (2012) proposes measuring the spread relative to the option's optionality.

Using high-frequency intraday data from 2004 to 2021 on U.S. equity options, we address the issues outlined at the outset of the introduction and compare the traditional relative bid-ask spread, the option liquidity measures proposed by the literature, and our new EAS along three key dimensions that are important for both researchers and practitioners:

(1) correlation with economic fundamentals: We evaluate the measures' sensitivity to economic fundamentals and market stress. The EAS demonstrates superior sensitivity compared to alternative measures. During periods of liquidity stress, such as the 2008 financial crisis or the COVID-19 pandemic, the EAS effectively highlights rising transaction costs, while other measures often understate or fail to capture these liquidity shocks. Additionally, the EAS exhibits high and statistically significant time-series correlations with key liquidity drivers. For example, the time series correlation with the underlying asset's bid-ask spread is 0.90, with the VIX it is 0.80, and with the TED spread 0.47. In contrast, the corresponding time-series correlations of other liquidity measures are lower, often statistically insignificant, and sometimes even counterintuitively negative. For example, the respective time-series correlations of the traditional relative bid-ask spread with the bid-ask spread of the underlying, VIX, and TED are 0.56, 0.20, and essentially 0. Also in cross-sectional analyses, the EAS has the strongest

<sup>&</sup>lt;sup>1</sup>From a technical point of view, consistency to bid-ask spreads in stock markets is established since stocks have a  $\Delta = 1$ .

correlations with key liquidity drivers, outperforming all other measures. For example, its crosssectional correlation with the underlying asset's bid-ask spread is 0.59, with the underlying's market capitalization -0.57, and with implied volatility 0.34. This compare to 0.42, -0.43, and an economically counterintuitive -0.09 for the relative quoted bid-ask spread. Interestingly, many liquidity measures specifically designed for options markets also show counterintuitive correlations with fundamentals. This observation applies particularly for the cross-sectional correlation with the option's implied volatility, which is negative for all liquidity measures, except for the difference between the ask and the bid implied volatility and the EAS. The counterintuitive relation with implied volatility is the result of the normalization, for example, with the option price or the IV. Importantly, all time-series and cross-sectional correlations of the EAS with key liquidity drivers are similar for calls and puts, and comparable to those observed for the underlying asset's bid-ask spread.

(2) consistency with market intuition: In line with market participants' expectations, liquidity measures should not imply that ITM options, which are traded least frequently, are significantly more liquid than ATM or OTM options. In our sample, the average relative quoted spread of ITM call options is 5.5%, compared to 8.1% for ATM call options, and even 17.9% for OTM call options. Importantly, all liquidity measures designed for the options market resolve this inconsistency of the relative quoted spread. For example, the EAS ranks ATM and ITM options as similarly liquid, with average spreads slightly above 0.5%, while OTM options exhibit higher spreads of 0.85%. Consequently, the discrepancies in liquidity levels across moneyness categories are much smaller when measured by the EAS than by the relative quoted spread.

(3) robustness to sampling variations: Liquidity measures should not be overly sensitive to small changes in the sample. Many studies select samples based on the option's moneyness, either monthly (using the moneyness on the last trading day of the previous month) or daily. If choosing one sampling frequency over the other leads to large differences in the measured liquidity, comparing results across studies becomes challenging. We find that due to its strong sensitivity to small variations in leverage, the traditional relative quoted spread shows only moderate time-series and cross-sectional correlation between the liquidity of the two samples. For example, for ATM call options, the time-series correlation is 0.64 and the cross-sectional correlation is 0.68. Liquidity measures specifically designed for the options market tend to perform more consistently – especially for OTM options. The EAS once again shows the highest consistency with cross-sectional and time-series correlations of 0.9 or above in all comparisons. In contrast, the absolute bid-ask spread measured in terms of implied volatility, while performing well regarding its correlations with economic fundamentals, lacks robustness with respect to sample selection. In summary, the EAS is the only measure that consistently satisfies all three key dimensions we use to assess suitable liquidity measures for the options market.

Because the calculation of the EAS and other option-specific liquidity measures from intraday trade data requires the computation of deltas and implied volatilities and, therefore, is computationally burdensome, we first analyze three simple approximation methods for which we do not need to solve for the implied volatility and delta of bid, ask, mid, or trade prices in a binomial tree. We show that approximating the tree-implied quantities with the Black and Scholes (1973) formula either accounting for or ignoring dividend payments and an approximation using a Taylor expansion produce reliable results with correlations to the exact measures of more than 0.99.

Second, low-frequency approximations of liquidity measures offer a computationally efficient and data-accessible alternative to high-frequency metrics, particularly valuable when intraday trade data is scarce or when conducting large-scale studies spanning extended time periods. In our analysis, we explore the viability of low-frequency proxies in approximating high-frequency liquidity measures, focusing on their ability to capture both cross-sectional and time-series variations in option liquidity. We therefore calculate variants of our liquidity measures based on daily data, using simplified delta and implied volatility estimations. Our results demonstrate that these low-frequency proxies maintain strong correlations with their high-frequency counterparts, especially for the low-frequency version of the EAS. The correlation remains above 0.9 across most option moneyness categories, ensuring that relative liquidity rankings and timeseries trends are well-preserved.

Overall, our empirical tests underscore that the EAS outperforms standard benchmarks in both cross-sectional and time-series dimensions. For instance, during the 2008 financial crisis and the COVID-19 pandemic, EAS reveals more pronounced liquidity spikes than classic relative spread measures, aligning closely with underlying market volatility and funding conditions. Moreover, we find that EAS is highly correlated – often exceeding 0.90 – with recognized fundamentals like the underlying asset's bid-ask spread and implied volatility, highlighting its reliability as a measure of transaction costs. In addition, a computationally simpler, lowfrequency variant of the measure preserves the majority of these benefits.

Beyond its academic importance, our new elasticity-adjusted liquidity measure offers practical benefits for traders, market makers, and institutional investors who rely on accurate assessments of transaction costs. By explicitly tying transaction costs to the size of the underlying hedge, the EAS reflects the true economic exposure of an option position more clearly than standard measures. This feature enables practitioners to more reliably compare the liquidity of contracts across different strikes and maturities, anticipate potential slippage when adjusting hedges, and improve execution strategies. Moreover, EAS can be readily integrated into trading algorithms and risk management tools – helping market participants identify the most cost-effective points of entry or exit, monitor changes in hedging costs during stress periods, and fine-tune portfolios.

Liquidity is a fundamental concept across all financial markets, and its measurement has been widely studied in asset classes such as equities, bonds, and foreign exchange. In equity markets, seminal works like Amihud (2002) and Pástor and Stambaugh (2003) introduce price impact measures, highlighting the role of illiquidity in explaining stock returns and market anomalies. Bond markets, on the other hand, face additional complexities due to lower trading frequency and heterogeneity in bond characteristics. Schestag et al. (2016) show that different transaction cost measures are suitable for assessing liquidity in corporate bond markets, while Lin et al. (2011) emphasize the role of liquidity risk in expected bond returns. In the foreign exchange market, liquidity is often proxied using order flow and bid-ask spreads, with studies like Ranaldo and de Magistris (2022) link liquidity to macroeconomic fundamentals and market volatility. Commodity markets have also attracted attention, with works such as Brunetti et al. (2016) demonstrating the importance of liquidity in futures markets and its sensitivity to economic uncertainty. While these studies provide key insights into liquidity dynamics, the unique characteristics of options – such as embedded leverage and varying moneyness – make traditional measures insufficient, necessitating specialized approaches like the one proposed in this paper.

The study of options market liquidity has received significant attention in recent years, with researchers proposing various measures to capture the complexities of transaction costs in options markets. Traditional approaches often adapt equity market measures, such as the relative quoted spread (Goyenko et al., 2009), to options (e.g., Muravyev and Pearson, 2020), but these methods can overlook the embedded leverage and moneyness effects unique to options. Normalizing the spread by optionality (Grundy et al., 2012) or by the option's volatility-adjusted price (Chaudhury, 2015) enhances comparability across different option strikes. More recently, Hsieh and Jarrow (2019) advocate implied-volatility-based spreads, arguing that implied volatilities are less sensitive to moneyness. Despite these advances, many existing measures fail to capture liquidity dynamics during market stress – such as the 2008 financial crisis or the COVID-19 pandemic. While Engle and Neri (2010) and Liu et al. (2018) highlight links between option illiquidity and economic fundamentals (e.g., volatility, funding constraints), they leave inconsistencies across strikes unresolved. All existing methods do not reflect the direct linkage between replication costs and underlying hedging exposures. Our Elasticity-Adjusted Spread (EAS) bridges this gap by scaling the quoted spread through the option's delta and underlying price, thereby pinpointing how much of the underlying asset must be hedged. In our empirical analvsis, this measure not only flags liquidity crises more clearly but also shows higher correlations with market fundamentals – indicating that EAS more accurately captures the true economic costs embedded in option transactions.

## 1 Data

Our analysis is based on two major data sources, CBOE LiveVol and IvyDB OptionMetrics, and covers the period from January 1, 2004 to June 30, 2021. To construct our high-frequency measures, we merge the trade-level data from LiveVol, which contains all options trades on all U.S. exchanges, with the daily data from OptionMetrics using the unique key specified by trade date, expiration date, strike price, option type, and root symbol and restrict our sample to equity options. Full details on the merging and filtering process are described in Appendix A.

We filter the merged data based on a two-step approach. In the first step, for which we provide an overview in Panel A of Table 1, we apply a set of basic error filters commonly used in the literature. These filters reduce the sample by 2.63%, of which the largest part (about 1.5% of the total number of observations) have either a negative bid-ask spread or a bid price of zero. Based on the filtered data set, we select our option samples for short-term puts and calls and three different moneyness-categories. On each trading day, the selection is based on OptionMetrics data from the previous day to rule out look-ahead bias (see Duarte et al., 2023). A call option is treated as OTM if  $0.125 < \Delta \leq 0.375$ , ATM if  $0.375 < \Delta \leq 0.625$ , or ITM if  $0.625 < \Delta \leq 0.875$ , while a put option is treated as ITM if  $-0.875 < \Delta \leq -0.625$ , ATM if  $-0.625 < \Delta \leq -0.375$ , or OTM if  $-0.375 < \Delta \leq -0.125$ . To minimize the impact of illiquid options, we only select plain vanilla short-term options that have a positive open interest at the time of selection, expire on the third Friday of the following month, and restrict our sample to S&P500 underlyings. In the second filtering step, for which we provide an overview in Panel B of Table 1, we apply filters that are related to the calculation of implied volatilities (IVs).<sup>2</sup> Thus, we exclude observations on days when the zero rate in OptionMetrics is missing, and for underlyings that have non-standard distributions or more than one dividend payment until the expiration of the option. We also drop observations for which we cannot calculate the mid IV as the mid price violates arbitrage bounds as well as observations with unrealistic underlying prices. Finally, for calculating the liquidity measures, we include option trade observations only for underlyings that exhibit at least eight observation days within a given month. The final samples are summarized in the last rows of Panel B in Table 1. We see that the samples for call options are larger than those for put options, reflecting their higher trading volume. In addition, OTM and ATM options were traded much more frequently than ITM options in our samples. On average, about 390 underlyings per month remain in the samples for OTM and ATM options. Again, the values for ITM options are significantly lower with 343 underlyings for calls and 253 underlyings for puts.

Table 2 reports summary statistics for the average option series and trades in our sample,

<sup>&</sup>lt;sup>2</sup>Note that many of our liquidity measures use IVs and deltas as inputs. We calculate these quantities using binomial trees (for puts and for calls with dividends) or the Black-Scholes formula (for calls without dividends).

calculated from the daily trade-weighted average per option series. Statistics are shown separately for calls and puts, aggregated over moneyness. On average, a call option series has a strike price of \$135, expires in 33 days and is traded 50 times per day. An average call option trade has a price of \$4.10, an implied volatility of 35%, a size of 16 contracts (or \$3,015), and occurs when the underlying price is \$132 and the delta is 0.44. For puts the magnitudes are similar.

## 2 Suitable measures of option liquidity

The aim of this section is to answer the question of how to optimally measure option liquidity when intraday trade data is available. We first discuss in Section 2.1 desirable characteristics of liquidity measures. Section 2.2 then reviews the approaches currently used in the literature to measure liquidity and suggests a new approach: elasticity-adjusted bid-ask spreads. In Section 2.3, we then compare the liquidity measures along the criteria from Section 2.1.

## 2.1 How to identify suitable liquidity measures?

Liquidity measures that are used in stock markets and most other asset markets are usually based on normalized transaction costs. To calculate them, a measure of (dollar) transactions costs, like the quoted or effective bid-ask spread, is divided by the mid price of the asset. In options markets, there is a significant price difference between options depending on their moneyness, primarily due to the inherent leverage of options. In-the-money (ITM) options, with their low leverage, have significantly higher prices than out-of-the-money (OTM) options, which have high leverage. Therefore, the normalization with the mid price is problematic and mainly contributes to the varying behavior of the quoted spreads in the daily and monthly selected samples, as shown in Panel B of Figure 1. In essence, standard transaction cost measures like the relative quoted (effective) bid-ask spread not only depend on the liquidity of the options but are also strongly affected by their inherent leverage.

Motivated by the problems when adapting the traditional relative quoted (and effective) spread to the options market, we propose three criteria to identify appropriate option liquidity measures:

First, appropriate liquidity measures should be related to economic fundamentals. This means, on the one hand, that liquidity stress periods like the financial crisis of 2008 or the COVID pandemic of 2020 should be visible in the liquidity measure's time series. On the other hand, option liquidity measures should exhibit positive relations with measures that are known to be related to asset liquidity in other markets like the VIX or the the TED spread (see e.g.,

Nagel, 2012; Brennan et al., 2012; Liu et al., 2018). In the cross-section, we expect suitable liquidity measures to be related to the liquidity of the underlying, the option's implied volatility, and the underlying's market capitalization (see, e.g., Fang et al., 2009; Engle and Neri, 2010; Brennan et al., 2012; Goyenko et al., 2015).

Second, it would be preferable if a liquidity measure would also address, at least to a certain extent, the observation from Figure 1 that ITM options that are traded least seem to be significantly more liquid than ATM and OTM options.

Third, liquidity measures should have a low dependence on small variations in the sample. For example, if an option sample is created based on the options' moneyness on the previous day, we want its average liquidity to be comparable to a similar sample that is created based on the moneyness at the end of the previous month. If this criterion is not fulfilled, it is not possible to compare scientific results that are sometimes based on daily and sometimes based on monthly selection criteria. Moreover, if slight variations in the sample construction lead to strong variations in the liquidity measures, robust results are hard to obtain in any study.

## 2.2 Definition of option liquidity measures

The drawbacks of standard liquidity measures applied to options markets have been documented before (see, e.g., Grundy et al., 2012; Chaudhury, 2015; Muravyev and Pearson, 2020). Based on these problems, researchers have developed adaptions of old measures or have proposed completely new liquidity measures, but there is up to this paper no comparative analysis of the different measures and approaches.

The methods proposed in the literature to overcome the problems of the standard liquidity measures can be divided into two general categories. The first approach is based on the dual relation between the option price and the associated implied volatility. Thus, bid-ask spreads are computed based on the difference between the implied volatilities obtained from bid and ask prices (see, e.g., Hsieh and Jarrow, 2019). Using implied volatilities rather than price levels has the advantage that implied volatilities are much less sensitive to the moneyness (and thus leverage) of the option. The second approach is to calculate the bid-ask spread not relative to the option price, but relative to some alternative metric that has been adjusted for leverage. For example, Grundy et al. (2012) use the option's optionality for normalization.

The literature proposes three measures that are using the first approach and calculate the bid-ask spread based on implied volatilities. The first measure, which is proposed by Chaudhury (2015), substitutes bid, ask, and mid prices with their respective implied volatilities. We term this measures relative implied volatility quoted spread and calculate it as

Relative IV QS = 
$$\frac{IV^{Ask} - IV^{Bid}}{IV^{Mid}}$$
, (2)

where  $IV^{Ask}$  denotes the implied volatility that corresponds to the ask price of the option,  $IV^{Bid}$  is the implied volatility that corresponds to the bid price, and  $IV^{Mid}$  is the implied volatility that corresponds to the midpoint of the bid and the ask price.<sup>3</sup>

A variant of the relative implied volatility quoted spread that is used by Hsieh and Jarrow (2019), divides the absolute implied volatility quoted spread by the level of the realized volatility of the underlying of the corresponding contract period. Thus, the resulting measure is

Hsieh & Jarrow IV QS = 
$$\frac{IV^{Ask} - IV^{Bid}}{RV_S}$$
, (3)

where  $RV_S$  denotes the aforementioned realized volatility of the underlying.

As the implied volatility level of an option should be much more uniform and therefore better comparable across different options than the price levels, it may even be sufficient to examine the absolute IV quoted spread as in Hsieh and Jarrow (2019),

Absolute IV QS = 
$$IV^{Ask} - IV^{Bid}$$
. (4)

There are two measures that follow the second approach of measuring the (dollar) bidask spread relative to alternative quantities that somehow account for the leverage of the option. Thus, as the fourth measure, we consider Chaudhury's (2015) "spread relative to dollar volatility" (hereafter called Chaudhury's measure), which is given by

Chaudhury QS = 
$$\frac{O^{Ask} - O^{Bid}}{S^{Mid} \cdot IV^{Mid} \cdot \sqrt{1/252}},$$
 (5)

where  $O^{Ask}$  and  $O^{Bid}$  denote the ask and the bid price of the option, and  $S^{Mid}$  is the midpoint of the bid and ask price of the underlying. Instead of the price itself, this measure scales the dollar spread by the two primary determinants of the option price: the underlying price and the implied volatility. The new normalization quantity can be interpreted as the (expected) one standard deviation dollar price change of the option over the next trading day, allowing for a more meaningful comparison across options written on different underlyings.<sup>4</sup>

The fifth measure is the spread relative to optionality

QS rel. optionality = 
$$\frac{O^{Ask} - O^{Bid}}{O^{Mid} - \max\{intrinsic\_value, PV(forward)\}},$$
(6)

<sup>&</sup>lt;sup>3</sup>Following Engle and Neri (2010), we restrict our implied volatilities to stay in the interval [0.001, 9]. If a bid or ask IV is out of this interval or the corresponding prices violate arbitrage bounds, we set the IV to the appropriate bound of the interval. For further details on the calculation of the implied volatilities, see Appendices A and B.

<sup>&</sup>lt;sup>4</sup>To calculate a daily volatility, Chaudhury (2015) divides the (per annum) implied volatility by the square root of the number of trading days in a year, which on average is 252 days in the U.S.

of Grundy et al. (2012), where *intrinsic\_value* denotes the intrinsic value of the option,

$$PV(forward) = \begin{cases} S - PV(div) - Ke^{-rT} & \text{for calls,} \\ Ke^{-rT} - S + PV(div) & \text{for puts,} \end{cases}$$
(7)

is the present value of a forward contract that is equivalently specified to the option and PV(div) denotes the present value of the dividend payments of the underlying.<sup>5</sup> Grundy et al. (2012) motivate their measure as calculating the dollar spread of the option relative to the value of the option above its lower bound, which is given by the maximum of the value of immediate exercise (*intrinsic\_value*) and the present value of committing to exercise at the maturity date of the option (PV(forward))). This excess value reflects the value of the asymmetric payoff profile of the option, making options of different moneyness more easily comparable and allowing for a more meaningful assessment of the option's liquidity.

Surprisingly, to the best of our knowledge, there is up to this paper no liquidity measure that uses more straightforward approaches to correct for the embedded leverage of an option. Therefore, we propose an elasticity-adjusted spread where we multiply the standard relative spread with the absolute value of the inverse of the option's elasticity  $E = \frac{\partial O}{\partial S} \frac{S}{O} \equiv \Delta \frac{S}{O}$ , where O and  $\Delta$  denote the price and the delta of the option, and S the price of its underlying. The option elasticity is the most direct measure for leverage (see, e.g., Frazzini and Pedersen, 2022).

Normalizing the traditional relative bid-ask spread with the elasticity leads to the already introduced Equation (1), which can be refined for quoted spreads as follows:

Elasticity adj. QS := 
$$\left| \frac{1}{E_{0.1}^{Mid}} \right| \frac{O^{Ask} - O^{Bid}}{O^{Mid}} = \left| \frac{1}{\Delta_{0.1}^{Mid} \frac{S^{Mid}}{O^{Mid}}} \right| \frac{O^{Ask} - O^{Bid}}{O^{Mid}}$$

$$= \frac{O^{Ask} - O^{Bid}}{|\Delta_{0.1}^{Mid}|S^{Mid}},$$
(8)

where

$$\left|\Delta_{0.1}^{Mid}\right| := \max\left\{|\Delta^{Mid}|, 0.1\right\}$$
(9)

denotes a caped version of the option's mid delta.<sup>6</sup> Note that Equation (8) can also be interpreted as the dollar bid-ask spread normalized with the exposure of the option to the absolute value of the replicating portfolio ( $|\Delta| \cdot S$ ). As stocks have a  $\Delta = 1$ , an advantage of our new measure over the other proposed measures is that for a stock, the Elasticity adj. QS equals the traditional relative quoted spread. Hence, this measure is the first that not only enables a meaningful liquidity comparison between options, but also allows to compare liquidity between

 $<sup>{}^{5}</sup>$ Grundy et al. (2012) remove all observations where the QS rel. optionality exceeds 0.5. In our analysis, we keep these observations, but set values that exceed 0.5 to 0.5 so that we have a full sample for all liquidity measures.

<sup>&</sup>lt;sup>6</sup>The main reason of restricting the delta of the option is to avoid having very small numbers in the denominator.

options and underlying markets.

All liquidity measures can also be calculated based on the effective spread, rather than the quoted spread, by using  $2 |O^{Trade} - O^{Mid}|$  instead of the difference between the quoted ask and bid price in the numerator. For IV based measures, we use  $2 |IV^{Trade} - IV^{Mid}|$ .

Throughout the entire analysis, we always consider the monthly averages of the liquidity measures. To do so, we first calculate the measure for each trade. We then form the daily equally weighted average across all transactions in all option series for a given underlying.<sup>7</sup> To obtain a monthly measure, we calculate the equally weighted average of all daily measures per underlying and month. Finally, we winsorize liquidity measures at the 1% and the 99% level on a monthly basis to mitigate potential outliers. Panel A of Table 3 summarizes the liquidity levels and standard deviations of the quoted-spread based liquidity measures that are based on intraday trade data.

### 2.3 Results

### **Requirement 1: Correlation with economic fundamentals**

As discussed in Section 2.1, we expect suitable liquidity measures to be related to economic fundamentals so that liquidity stress periods are visible from their time series. Panel B of Figure 1 and Panel A of Figure 2 present the time series of average liquidity for options. During the 2008 financial crisis and the COVID turmoil in 2020, only the Elasticity adj. QS and, to some extent, the Absolute IV QS indicate significantly higher transaction costs compared to periods of low market stress. The differences between the measures become most obvious when we compare them with the quoted spread of the underlyings in Panel B of Figure 2. All high-frequency measures except the Elasticity adj. QS exhibit a markedly different behavior compared to this time series.

To formally analyze the relations between liquidity measures and fundamentals, we calculate time-series and cross-sectional correlations between the high-frequency liquidity measures and variables that are known to be related to liquidity from other studies (see Section 2.1). Panel A of Table 4 presents time-series correlations between liquidity measures and fundamentals for ATM options.<sup>8</sup> The highest correlation with a high-frequency liquidity measure is highlighted with a solid box; dashed boxes indicate correlations that are not significantly different from the highest correlation in the same column. Correlations that significantly deviate from zero

 $<sup>^{7}</sup>$ Contract volume weighting produces results that are qualitatively and quantitatively similar to those of the equally weighted average. Consequently, we limit our presentation of results to the latter.

<sup>&</sup>lt;sup>8</sup>We first calculate the monthly mean of liquidity across all underlyings and then compute the correlations between the time series of liquidity and the respective fundamental.

at the 5% level are shown in bold font.<sup>9</sup> Both the Elasticity adj. QS and the Absolute IV QS are very strongly correlated to the Underlying QS with a correlation of above 0.9, respectively. In contrast, the correlation of the other liquidity measures with the Underlying QS is always below 0.6 and even insignificantly different from zero for the Hsieh & Jarrow QS. Regarding the other three variables, implied volatility of the options, the VIX, and the TED spread, we also expect positive correlations with option liquidity. A good reference point are the correlations of these variables with the Underlying QS, which we present in the last row of Panel A in Table 4. However, the only two liquidity measures that show a positively significant correlation with all three stress variables are the Elasticity adj. QS and the Absolute IV QS. The other measures yield only low or even negative correlations with some of the stress variables.

Average cross-sectional correlations with the underlying QS, the implied volatility, and underlying market capitalization are shown in Panel B of Table 4.<sup>10</sup> Again, the Elasticity adj. QS (and the Absolute IV QS) show the absolutely highest average cross-sectional correlations with all three possible drivers of option liquidity. For the other option liquidity measures, the correlations with the Underlying QS are considerably lower but positive and significantly different from zero. This is in contrast to the correlations with the implied volatility, which even exhibit a counter-intuitive negative sign for all but the Elasticity adj. QS and the Absolute IV QS. The negative correlations range from -0.04 to -0.13 and are significantly different from zero.<sup>11</sup> Finally, we observe statistically significant negative cross-sectional correlations between the high-frequency measures and the end-of-month market capitalization of the underlying, ranging for call (put) options from about -0.39 (-0.38) for the Hsieh & Jarrow IV QS to -0.57 (-0.55) for the Elasticity adj. QS. Interestingly, the correlations of the Elasticity adj. QS and the Absolute IV QS with implied volatility and market capitalization of the underlying are again on the same order of magnitude as the corresponding correlations of the Underlying QS which we show in the last row of this table.

<sup>&</sup>lt;sup>9</sup>We follow Goyenko et al. (2009) and use the test statistic  $\rho \sqrt{\frac{n-2}{1-\rho^2}}$  to test for the sample size *n* whether timeseries correlations  $\rho$  are significantly different from zero at the 5% level. To test whether two measures differ significantly at the 5% level, we use Steiger's Z test for correlations of dependent samples and non-overlapping variables (see Steiger, 1980).

<sup>&</sup>lt;sup>10</sup>Average cross-sectional correlations are determined by first calculating cross-sectional correlations across all underlyings for each month, subsequently applying Fisher's Z-transformation, taking the mean and transforming the results back. In the cross-sectional analysis, we follow Goyenko et al. (2009) and test the average cross-sectional correlations against zero by running a *t*-test and test for differences between the correlations of two high-frequency measures by *t*-testing the mean of the differences of the monthly Z-transformed correlations. For all tests, we use a Newey and West (1987) correction with four lags to compute the standard errors.

<sup>&</sup>lt;sup>11</sup>This pattern was already observed for the IV based relative quoted spread by Hsieh and Jarrow (2019).

#### **Requirement 2:** Consistency with market intuition

Contrary to the general perception of market participants, standard liquidity measures often indicate that ITM options, despite of being least traded, are significantly more liquid than ATM and OTM options. Suitable liquidity measures for the options market should address this counterintuitive pattern. To evaluate this, we compare the average levels of the measures in Panel A of Table 3.

We observe similar liquidity levels for calls and puts within the same moneyness category. However, the relative order of liquidity according to moneyness does not yield a consistent picture across the different high-frequency measures. Depending on the measure employed, it is possible for either OTM, ATM, or ITM options to appear most liquid. When utilizing the traditional quoted spread, ITM options exhibit the lowest spreads. Conversely, measures based on implied volatility indicate that ATM and OTM options are similarly liquid to each other and more liquid than ITM options. The QS rel. optionality suggests ATM options, while the Chaudhury QS indicates OTM options as the most liquid. Regarding the Elasticity adj. QS, both ATM and ITM options show low spreads compared to OTM options. Importantly, all six measures that account for the options' embedded leverage resolve the puzzling observation that ITM options appear to be by far the most liquid options using the traditional quoted bid-ask spread, and, therefore, fulfill the second requirement discussed in Section 2.1.

#### **Requirement 3: Robustness to sampling variations**

As discussed in Section 2.1, it is problematic for any liquidity measure if small variations in the sample can lead to large effects on the average liquidity. To investigate the stability of liquidity measures, we design a test that compares average liquidity when the samples are constructed based on the moneyness on the previous trading day with analogously constructed samples based on the moneyness on the last day of the previous month.<sup>12</sup> Both of these approaches are prevalent in the option literature (e.g., Muravyev and Pearson (2020) and Duarte et al. (2022) rebalance their portfolios on a daily basis, whereas Bollen and Whaley (2004) and Frazzini and Pedersen (2022) use a monthly rebalancing).

For an initial intuition of the measures' dependence on small variations in the sample, we visually inspect their time series across the different samples for ATM call options. The time series of the standard quoted spread measure in Panel B of Figure 1 is extremely noisy and hardly resembles the time series of the daily selected sample. Looking at the behavior of our six option liquidity measures in Panel A of Figure 2, we observe that the Relative IV QS, the Absolute IV QS, the Hsieh & Jarrow IV QS, and the QS rel. optionality show only limited

 $<sup>^{12}</sup>$ All filters are applied analogously to the daily selected samples as described in Panel B of Table 1.

progress compared to the standard quoted spread measure, with the QS rel. optionality even indicating entirely different levels of liquidity. In contrast, the Chaudhury QS and, in particular, the Elasticity adj. QS are clearly more stable and show a very consistent behavior across the daily and monthly selected samples.

More formally, we compare time-series and cross-sectional correlations between the liquidity of the daily and monthly selected samples. The correlations are computed separately for each high-frequency measure, each moneyness category, and for call and put options. The time-series correlations are presented in Panel A of Table 5. For almost all sample pairs (except for OTM calls), the Elasticity adj. QS yields the highest correlations, exceeding 0.9. For OTM call options, the Absolute IV QS and the Chaudhury QS perform slightly better, but the differences in the correlations are less than 0.015. For OTM options, all alternative high-frequency measures perform better than the traditional quoted spread. For ITM options, the traditional quoted spread yields high correlations between the daily and monthly selected samples and is even the second-best measure for put options. The good performance of the traditional quoted spread is likely due to the fact that ITM options are least affected by embedded leverage. In contrast, differences in the correlations are strongest for ATM options, which are most affected by slight variations in the moneyness. For these options, none of the other measures comes close to the Elasticity adj. QS.

Panel B of Table 5 shows the average cross-sectional correlations. As for the time-series correlations in Panel A, the Elasticity adj. QS is the only high-frequency measure that consistently shows high correlations exceeding 0.9 between the daily and monthly selected samples. Only for OTM options, the Chaudhury QS yields slightly higher correlations. For these options, all alternative high-frequency measures perform better than the traditional quoted spread in the cross-sectional analysis. Conversely, for ATM and ITM options, the high-frequency measures based on implied volatility perform worse than the traditional quoted spread.

In summary, the Elasticity adj. QS is the most suitable high-frequency measure of option liquidity since it is the only measure that performs consistently well regarding all three keydimensions we use to assess suitable liquidity measures: It not only partially resolves the problem that ITM options seem to be the most liquid according to traditional liquidity measures, but also shows a consistent behavior for small variations in the sample selection, allowing for meaningful comparisons of liquidity over time and across different samples and, by construction, even between options and stocks. This measure also clearly identifies periods of high market stress like the financial crisis and the COVID pandemic and yields the highest correlations with different drivers of option liquidity. All other measures fail at least in one dimension, with the Absolute IV QS being the most promising alternative. However, the Absolute IV QS, which has only slightly lower correlations with economic fundamentals than the Elasticity adj. QS, lacks robustness with respect to sample selection, especially for ITM options. All other measures are, for different reasons and in different settings, not suitable to measure option liquidity. The results for effective-spread-based measures are qualitatively similar to those of the quoted spread and are presented in Appendix C.2.

## 3 Approximation methods

The measures analyzed in Section 2 are costly to compute for two different reasons. First, they require intraday trade data, which is not easily available and which requires some effort to clean and process it. Second, the measures that incorporate leverage require implied volatilities for the ask, bid, or mid price or the delta of the option at the time of trading. As equity options are of American type, a potential early exercise must be taken into account, requiring the use of numerical methods to calculate these quantities.

In Section 3.1, we analyze and compare approximations that reduce the computational effort to calculate implied volatilities and deltas, Section 3.2 then compares our high-frequency measures with low-frequency approximations that only require daily data. The methodology for these analyses follows the standard procedures described in the literature (see, e.g., Goyenko et al., 2009; Schestag et al., 2016). We compare both time-series and average cross-sectional correlations, as well as the mean bias and RMSE between the proxy measure and the corresponding benchmark.

### 3.1 Approximations for IV and delta

A major part of the computational effort of the high-frequency measures arises from the calculation of the implied volatilities or the delta of the option using a binomial tree, which requires a large number of time steps to get an accurate result. We analyze two approaches to circumvent the time-consuming evaluation of the binomial tree: the first is to approximate the price of the American option with the Black and Scholes (1973) formula, which is then inverted using numerical methods to determine the implied volatility. The second is the direct approximation of the implied volatility or delta using a Taylor expansion.

A straight-forward way to approximate the binomial-tree implied volatility and delta is to disregard the possibility of early exercise and employ the Black and Scholes (1973) formula. Doing so, it is possible to account for dividends by subtracting the present value of the dividend payment from the underlying price and using the result as an input for the Black and Scholes (1973) formula. This could lead to a better approximation of the corresponding implied volatility compared to simply ignoring the dividends, but involves additional programming effort and runtime for dealing with the dividend data. Alternatively, we also analyze the possibility of not accounting for the dividend payments as a second approximation method.

The change in the intraday option price can be approximated with a Taylor expansion that uses the closing option price and the corresponding sensitivities from the previous trading day, e.g., from OptionMetrics by

$$dO = \Delta_{t-1} \cdot dS + \frac{1}{2} \Gamma_{t-1} \cdot (dS)^2 + Vega_{t-1} \cdot dIV, \qquad (10)$$

where  $d[\cdot]$  denotes the change in the respective variable from the end of the previous trading day to the time of the trade, O denotes the price of the option, S the price of the underlying, IV the implied volatility, and  $\Delta_{t-1}$ ,  $\Gamma_{t-1}$ , and  $Vega_{t-1}$  the closing delta, gamma, and vega of the option at the end of the previous trading day (see, e.g., Equation (11) in Büchner and Kelly, 2022). Solving Equation (10) for the change in the implied volatility dIV yields

$$dIV = \frac{dO - \Delta_{t-1} \cdot dS - \frac{1}{2}\Gamma_{t-1} \cdot (dS)^2}{Vega_{t-1}}$$
(11)

and we can approximate the implied volatility at the time of the trade with

$$IV_{trade}^{prox} = IV_{t-1} + dIV, \tag{12}$$

where  $IV_{t-1}$  is the closing implied volatility from the previous trading day. We further approximate the option's delta using the gamma-approximation

$$\Delta_{trade}^{prox} = \Delta_{t-1} + \Gamma_{t-1} \cdot dS. \tag{13}$$

Table 6 compares the results of using the Black and Scholes (1973) formula with and without dividends ('BS div' and 'BS std') and the Taylor expansion ('Taylor') to approximate our high-frequency measures for ATM options.<sup>13</sup> All three approximation methods yield very high correlations consistently larger than 0.99 for both the time-series analysis presented in Panel A and the cross-sectional analysis presented in Panel B. The approximation that employs the Black-Scholes formula without accounting for dividend payments produces the most accurate results for call options. For put options, it is the approximation that incorporates dividend payments into the Black-Scholes formula that yields the most precise approximations. Regarding the approximation errors, the relative mean biases presented in Panel C are quite small but always significantly different from zero.<sup>14</sup> All biases are in the range between -0.05 and +0.03, implying that the approximation methods on average under (over) estimate transaction by less than 5% (3%). 24 out of 30 biases are negative, which means that underestimation

<sup>&</sup>lt;sup>13</sup>Since the traditional quoted spread and the QS rel. optionality do neither involve the implied volatility nor the delta of the option, they are excluded from this analysis. Using OTM and ITM options yields similar results, which are omitted to conserve space.

<sup>&</sup>lt;sup>14</sup>To make the size of the mean bias and RMSE comparable, we normalize the approximation error by the sample mean of the corresponding high-frequency measure.

of transaction cost levels is more prevalent than overestimation. Finally, Panel D shows the relative RMSEs that are also always statistically significant but fairly low, implying average errors between 2% and 16%. For both types of approximation errors, using the Taylor expansion often yields slightly less accurate results than the two other methods. In summary, all three approximation methods work quite well which is also true when using effective-spread-based measures (see Appendix C.2 for details).

### 3.2 Low-frequency approximations

Using intraday trade data presents two potential challenges in practice. First, high-frequency data typically involves a vast quantity of information, especially in the options market, where numerous options with varying strike prices and maturities exist for each underlying asset. For example, the LiveVol raw dataset used in our analysis has a size of approximately 3.5 terabyte. Second, only daily data is often easily available for a long sample period and a broad cross section. In the spirit of Goyenko et al. (2009), we therefore analyze whether it is possible to approximate high-frequency liquidity measures using daily data.

We calculate four different low-frequency approximations analogously to their high-frequency benchmark measures, differing only in how the implied volatilities and deltas are calculated. The first version is an exact replication of the high-frequency measure calculated with lowfrequency data using a binomial tree to calculate the implied volatility ('Exact'). The other three low-frequency proxies follow the methods presented in Section 3.1 and approximate the implied volatilities using the Black and Scholes (1973) formula with or without incorporating dividend payments and a Taylor expansion. Note that for the traditional quoted spread and the QS rel. optionality measures, the three approximate methods for the IVs and delta are not necessary and we only calculate the exact replication with daily low-frequency data. In addition to the four proxies that are analogously calculated to the respective high-frequency measure, we employ three alternative low-frequency liquidity proxies that are commonly used in the literature: the Amihud (2002) measure, the Pástor and Stambaugh (2003) measure, and a low-frequency version of the traditional quoted spread.<sup>15</sup>

A crucial input variable for calculating the Amihud (2002) measure is the option's return, and for the Pástor and Stambaugh (2003) measure, the option's return and the market return of equity options. Since option prices (and thus returns) heavily depend on the embedded leverage of the option and changes in the underlying prices, we calculate delta-hedged and leverage-adjusted returns to achieve comparability across different option series. More precisely,

<sup>&</sup>lt;sup>15</sup>Note that both the proxies of Amihud (2002) and Pástor and Stambaugh (2003) measure price impact and, therefore, have different units than our high-frequency transaction cost measures. In the literature, they are often used as general proxies for liquidity (see, e.g., Agarwal et al. (2015) for stocks, Lin et al. (2011) for bonds, or Cao and Wei (2010); Choy and Wei (2020) for options).

we calculate the delta-hedged and leverage-adjusted option returns as

$$R_{hed\&lev} = \frac{1}{|E_{t-1}|} \frac{O_t - O_{t-1}}{O_{t-1}} + \left(1 - \frac{1}{|E_{t-1}|}\right) r_f \pm (R_S - r_f), \qquad (14)$$

following Choy and Wei (2020), where plus in the plus-minus sign corresponds to puts and minus to calls,  $E_{t-1}$  denotes the option's elasticity,  $R_S$  is the return of the underlying, and  $r_f$  is the risk-free rate.<sup>16</sup> To determine the daily market return of equity options, we take the equally weighted average of all hedged and leverage-adjusted individual option returns of all available standard equity option series, independent of maturity or moneyness.<sup>17</sup>

Table 7 presents the results of the comparison of the daily approximation methods for ATM call options. With regard to the time-series correlations in Panel A, all proxies that are analogous to their high-frequency counterparts yield high correlations above 0.89. The proxies of the Elasticity adj. QS, the winner in the high-frequency analysis in Section 2, produce the highest correlations of about 0.99. The proxies of the Absolute IV QS perform only slightly worse. For the other measures, the correlations between the proxies and their benchmark are lower and range between 0.89 and 0.91. The alternative proxies in the last three columns in Table 7 produce overall lower and more heterogeneous time-series correlations. Whereas the time-series correlations of the Amihud (2002) measure with the Elasticity adj. QS and the Absolute IV QS are with 0.91 to 0.92 quite high, its correlations with the other measures are much lower, and for the Hsieh & Jarrow IV QS even slightly negative. The Pástor and Stambaugh (2003) measure shows the lowest correlations ranging from 0.11 for the high-frequency Elasticity adj. QS to 0.21 for the high-frequency QS rel. optionality. The correlations of the Pástor and Stambaugh (2003) measure with the high-frequency versions of the Elasticity adj. QS and the Absolute IV QS are even statistically insignificant. The low-frequency version of the quoted spread produces moderate to high time-series correlations with the high-frequency measures ranging from 0.58 for the Hsieh & Jarrow IV QS to 0.91 for the high-frequency version of the Quoted Spread. However, the proxies that are calculated analogously to the high-frequency measures yield much better results.

The average cross-sectional correlations of the proxies that are analogously calculated to their high-frequency benchmarks provide a more homogeneous picture with correlations between 0.94 and 0.96. Again, the low-frequency versions of the Elasticity adj. QS produce the highest correlations while the Pástor and Stambaugh (2003) measure yields the lowest. The latter are slightly negative, but close to zero and often statistically insignificant. For the Amihud (2002) measure and the low-frequency version of the traditional quoted spread, we obtain qualitatively

<sup>&</sup>lt;sup>16</sup>We obtain the risk-free rate from Kenneth French's data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\_library.html).

<sup>&</sup>lt;sup>17</sup>We calculate the options market returns from the full OptionMetrics equity option universe to which we apply similar basic error filters as to the LiveVol trade data. For details on the filtering process, see Appendix A.

similar results as for the time-series correlations. The main difference is that the average crosssectional correlations of the Amihud (2002) measure with the Elasticity adj. QS and with the Absolute IV QS, although still higher than for the other high-frequency benchmarks, are with about 66% much lower than in the time-series.

In contrast to the high-frequency proxies in Table 6, the low-frequency proxies produce much higher estimation errors. The relative mean biases presented in Panel C of Table 7 are all statistically significant, positive, and range from 25% for the low-frequency proxy of the traditional quoted spread to 32% for the exact low-frequency proxy of the Absoulte IV QS. These large mean biases become even more obvious when comparing the liquidity levels of the high-frequency measures and the low-frequency proxies, presented in Table 3. The lowfrequency proxies show much higher bid-ask spreads than the corresponding high-frequency measures in all six moneyness samples. Consequently, using liquidity proxies calculated from closing prices for option liquidity introduces substantial biases, indicating much higher transaction costs than the high-frequency measures based on intraday trade data. These observations confirm the findings of Goyenko and Zhang (2021) that closing bid-ask spreads are much higher than the bid-ask spreads throughout the day and are consistent to the findings of Muravyev and Pearson (2020) that many market participants time their executions to minimize transaction costs. As a result of the high biases, the relative RMSEs in Panel D are also much higher than in Table 6 and range from 42% to 66%. Note that we cannot compare the levels of the alternative proxies with the high-frequency benchmarks in Panels C and D as the measures have different units. In summary, using low frequency approximations for measuring option liquidity works well, especially for the Elasticity adj. QS, as long only the variations in the time-series and cross-section are relevant. As soon as also correct liquidity levels are crucial, one has to take the bias from using daily closing prices into account. The results for ITM and OTM call options as well as ATM put options are similar and presented in Appendix C.1.

Low-frequency data usually are only available on a quote level, making it impossible to calculate effective-spread based proxies. We therefore analyze the capability of our quoted-spread-based low-frequency proxies to meaningfully approximate effective-spread-based high-frequency measures. We find that the Elasticity adj. QS is able to capture time series dynamics and cross-sectional variation of the Elasticity adj. ES, although its upward-bias is higher than when approximating the high-frequency Elasticity adj. QS. Detailed results for all measures are presented in Appendix C.2.

## 4 Conclusion

This paper presents a new framework for measuring option liquidity that addresses several limitations of existing measures. By comparing the existing liquidity measures used in options markets, we show that conventional liquidity metrics, such as the relative quoted spread or implied volatility-based measures, strongly depend on leverage and can produce counterintuitive results regarding their correlations with fundamentals and across moneyness categories. We design a new liquidity measure that resolves these shortcomings by normalizing the traditional bid-ask spread with the option's elasticity. Our empirical analysis demonstrates that the elasticity-adjusted spread correlates strongly with well-established liquidity drivers, including the underlying asset's bid-ask spread, implied volatility, and funding conditions, and it clearly highlights major market stress episodes such as the 2008 financial crisis and the onset of the COVID-19 pandemic.

These findings have two central implications. First, for researchers studying price formation, volatility, or return predictability in options markets, adopting an elasticity-based approach to liquidity measurement can reduce bias in empirical tests and highlight liquidity's role in explaining option returns. Second, market participants – particularly institutional investors and market makers – can benefit from more precise hedging cost estimates, and enhanced risk monitoring by incorporating elasticity-adjusted liquidity assessments into their trading models. Moreover, our study shows that computationally simpler, low-frequency approximations to this measure retain considerable accuracy for large-sample or long-horizon analyses, but it is important to take into account that intraday transaction costs are lower than transaction costs calculated from closing prices.

Future work could extend this framework to different derivative classes or international markets, examining whether idiosyncratic market microstructures and regulatory environments affect the performance of elasticity-based liquidity metrics. Additionally, exploring the dynamic interactions between option liquidity and limits to arbitrage, funding liquidity, and other systemic factors would offer further insight into how liquidity conditions evolve during extreme market events. In sum, our proposed measure not only advances the quantitative understanding of options market liquidity but also points to fruitful directions for ongoing research in asset pricing and market microstructure.

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Panel A: Comparison of moneyness categories by number of trades and quoted spread



Panel B: Quoted spread of ATM options depending on sample selection

Figure 1: Measurement problems of standard liquidity measures (call options). The quoted spread is defined as the difference of the quoted ask and bid price relative to the mid price and is calculated from LiveVol trade data on a monthly basis. Moneyness is defined by the absolute value of the delta: A call option is treated as OTM if  $0.125 < \Delta \le 0.375$ , ATM if  $0.375 < \Delta \le 0.625$ , or ITM if  $0.625 < \Delta \le 0.875$ .

The daily sample selection takes place at the end of the previous trading day, the monthly sample selection at

the end of the previous month. The observation period is from January 1, 2004 to June 30, 2021.



Panel A: Option liquidity



Panel B: Stock liquidity

# Figure 2: Dependence of leverage-adjusted liquidity measures on sample selection (ATM call options) and comparison to underlying liquidity.

Panel A shows high-frequency option liquidity measures and Panel B presents the quoted spread of the underlying at the time of the option trade. The high-frequency option liquidity measures are computed from LiveVol trade data and are described in Section 2.1. All measures are calculated on a monthly basis. ATM call options have a delta in the range of  $0.375 < \Delta \le 0.625$ . The daily sample selection takes place at the end of the previous trading day, the monthly sample selection at the end of the previous month. The observation period is from January 1, 2004 to June 30, 2021.

### Table 1: Overview on filters and final samples.

This table summarizes the filters applied to the raw data and the selected samples from the LiveVol data and provides an overview on the final samples. Panel A presents the filters applied to the full LiveVol data merged with OptionMetrics. The samples in Panel B are selected based on the option's moneyness in OptionMetrics on the previous trading day. Moneyness is defined by the absolute value of the delta at the end of the previous trading day: A call option is treated as OTM if  $0.125 < \Delta \le 0.375$ , ATM if  $0.375 < \Delta \le 0.625$ , or ITM if  $0.625 < \Delta \le 0.875$ , whereas a put option is treated as ITM if  $-0.875 < \Delta \le -0.625$ , ATM if  $-0.625 < \Delta \le -0.375$ , or OTM if  $-0.375 < \Delta \le -0.125$ . The final samples contain only plain-vanilla options that expire on the third Friday of the following month, have a positive open interest at time of the selection and are written on S&P500 members. All filters are described in the Appendix. The observation period is from January 1, 2004 to June 30, 2021.

Panel A: Basic error filters to the full LiveVol data (merged with OptionMetrics)								
	# Obs.	%						
Observations raw data	$3,\!414,\!560,\!360$	100.00%						
Outside trade hours	$692,\!343$	0.02%						
Negative spread or zero bid	51,283,173	1.50%						
Non-standard options	$3,\!621,\!355$	0.11%						
Trade price severely outside quote range	$15,\!476,\!303$	0.45%						
Huge deviation of bid and ask price	$10,\!526,\!477$	0.31%						
No arbitrage relations violated	18,045,396	0.53%						
All filters	89,713,724	2.63%						
Remaining observations after error filters	$3,\!324,\!846,\!636$	97.37%						

Panel B: Additional filters and final samples										
		Calls			Puts					
	OTM	ATM	ITM	OTM	ATM	ITM				
Observations after data selection	$95,\!375,\!455$	$95,\!275,\!182$	34,612,801	71,069,084	52,294,410	14,938,968				
Missing zero rate	39,949	51,277	20,520	$26,\!801$	26,346	9,554				
Non-standard distribution or more than one dividend	1,397,332	1,339,869	533,313	942,366	702,031	215,016				
Filters related to mid IV	16,261	33,469	88,147	3,231	12,699	54,133				
Filters related to underlying prices	316,146	416,474	$235,\!646$	261,275	270,382	138,865				
Less than eight obs. per underlying	489,004	793,419	488,409	346,871	569,586	614,190				
All additional filters	2,257,574	2,631,441	1,364,017	1,580,100	1,580,239	1,031,630				
Final sample	$93,\!117,\!881$	$92,\!643,\!741$	$33,\!248,\!784$	$69,\!488,\!984$	50,714,171	$13,\!907,\!338$				
# underlyings per month	398	391	343	392	363	253				
# option series per month	1,929	1,827	$1,\!621$	2,156	1,713	1,139				
# sec-months	83,589	82,188	71,984	82,361	76,190	53,168				

#### Table 2: Descriptive statistics for trades in our call and put samples.

This table presents time series averages calculated from the mean, standard deviation, and different percentiles of the cross sectional distribution over all call trades on a trading day in our sample. Embedded leverage is the ratio of underlying price to option price. Option midpoint and Underlying midpoint are the respective average of bid and ask prices at the time of the option trade. IV at trade price is the implied volatility that corresponds to the prevailing trade price of the option. Realized volatility is the realized (historical) volatility of the underlying of the corresponding contract period. Delta is the prevailing delta that corresponds to the option midpoint at the time of the option trade. Quoted Spread, Elasticity adj. QS and Absolute IV QS are described in Section 2.2. All liquidity measures are calculated at the time of the trade. Closing IV, Delta, Gamma and Vega are the closing metrics of the respective trading day as quoted in OptionMetrics. The observation period is from January 1, 2004 to June 30, 2021.

					Calls								Puts			
					I	Percentiles	3						I	Percentiles	3	
	Unit	Mean	St.Dev	5%	25%	50%	75%	95%	-	Mean	St.Dev	5%	25%	50%	75%	95%
Strike price	\$	135.37	195.93	17.29	38.22	71.01	150.44	514.12		132.77	188.83	17.68	37.95	70.12	144.94	515.48
Underlying midpoint	\$	132.14	191.04	16.80	37.33	69.49	148.15	499.78		136.87	194.60	18.20	38.91	72.26	150.26	531.21
Days-to-expiration	Days	32.68	8.94	18	24	32	39	46		32.68	8.94	18.00	24.00	32.00	39.00	46.00
Option trade price	\$	4.10	7.38	0.28	0.80	1.71	4.14	15.95		3.92	6.61	0.35	0.88	1.75	4.03	14.93
Trades	$1000 \mathrm{s/day}$	49.81	33.49	9.65	28.67	43.65	62.02	116.29		30.50	18.24	5.27	18.09	28.04	40.50	65.87
Trade size	Contracts	15.64	123.72	1.00	1.58	4.56	10.80	49.07		16.61	130.71	1.00	1.83	4.97	11.28	50.12
Dollar volume	1000s \$	3.15	24.57	0.07	0.27	0.78	2.22	10.56		3.37	29.14	0.08	0.31	0.86	2.37	10.78
IV at trade price	%	35.06	14.88	18.89	25.89	32.17	40.72	60.80		36.71	14.21	20.11	27.30	33.73	42.70	64.11
Realized volatility	%	34.14	16.53	16.29	23.39	30.28	40.61	65.29		34.24	16.57	16.30	23.45	30.34	40.70	65.84
Delta		0.44	0.18	0.16	0.30	0.43	0.56	0.77		0.39	0.17	0.15	0.25	0.37	0.50	0.72
Embedded Leverage		56.89	58.55	12.15	23.40	38.67	69.10	161.57		56.71	49.92	12.84	25.37	41.45	71.54	150.65
Relative Quoted Spread	%	5.33	7.37	0.50	1.65	3.10	6.07	17.36		5.02	6.64	0.50	1.66	3.06	5.79	15.84
Elasticity adj. QS	%	0.34	0.46	0.03	0.12	0.21	0.39	1.06		0.37	0.51	0.04	0.13	0.23	0.43	1.15
Absolute IV QS	%	1.42	2.32	0.13	0.46	0.84	1.59	4.33		1.42	2.56	0.13	0.45	0.83	1.56	4.24
Underlying QS	%	0.04	0.05	0.01	0.02	0.03	0.05	0.12		0.04	0.05	0.01	0.02	0.03	0.05	0.11
Absolute Quoted Spread	cents	11.26	20.75	1.09	2.68	5.17	11.61	40.47		11.24	20.07	1.07	2.73	5.33	11.83	39.93
Absolute Underlying QS	cents	5.78	14.98	0.80	1.00	1.33	4.20	25.74		6.20	15.77	0.80	1.01	1.42	4.65	27.79
Closing IV	%	34.65	13.00	18.88	25.82	32.06	40.58	59.90		36.58	14.14	20.06	27.21	33.61	42.58	63.95
Closing Delta		0.44	0.18	0.16	0.29	0.43	0.57	0.77		0.39	0.18	0.14	0.25	0.37	0.51	0.72
Closing Gamma		0.09	0.08	0.01	0.03	0.07	0.11	0.23		0.08	0.07	0.01	0.03	0.06	0.10	0.21
Closing Vega		13.45	20.02	1.59	3.72	6.90	14.71	50.47		13.50	19.69	1.70	3.82	6.99	14.53	51.24

### Table 3: Descriptive statistics for the high-frequency measures and corresponding low-frequency proxies.

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from Option-Metrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. Moneyness is defined by the absolute value of the delta at the end of the previous trading day: A call option is treated as OTM if  $0.125 < \Delta \le 0.375$ , ATM if  $0.375 < \Delta \le 0.625$ , or ITM if  $0.625 < \Delta \le 0.875$ , whereas a put option is treated as ITM if  $-0.875 < \Delta \le -0.625$ , ATM if  $-0.625 < \Delta \le -0.375$ , or OTM if  $-0.375 < \Delta \le -0.125$ . The observation period is from January 1, 2004 to June 30, 2021.

	Calls						Puts						
	ГО	ΓM	AT	M	IT	M		OT	'M	АТ	ΓM	IT	M
	Mean	SD	Mean	SD	Mean	SD	М	lean	SD	Mean	SD	Mean	$^{\mathrm{SD}}$
Panel A: High-frequency measures													
Quoted Spread	0.1796	0.1128	0.0816	0.0564	0.0549	0.0348	0.1	1467	0.0947	0.0700	0.0476	0.0473	0.0306
Relative IV QS	0.0770	0.0495	0.0763	0.0528	0.1578	0.1132	0.0	0680	0.0446	0.0734	0.0509	0.1645	0.1218
Hsieh & Jarrow IV QS	0.0850	0.0598	0.0870	0.0644	0.1934	0.1474	0.0	0865	0.0623	0.0840	0.0617	0.1744	0.1338
Absolute IV QS	0.0227	0.0202	0.0232	0.0211	0.0499	0.0419	0.0	0230	0.0208	0.0234	0.0240	0.0530	0.0627
QS rel. optionality	0.1662	0.0925	0.0971	0.0623	0.2321	0.0948	0.1	1395	0.0819	0.0929	0.0582	0.2459	0.0947
Chaudhury QS	0.1084	0.0716	0.1352	0.0919	0.1878	0.1191	0.0	0948	0.0635	0.1291	0.0871	0.1882	0.1248
Elasticity adj. QS	0.0085	0.0074	0.0054	0.0048	0.0052	0.0043	0.0	0087	0.0076	0.0054	0.0052	0.0053	0.0058
Panel B: Low-frequency proxies													
Quoted Spread	0.2491	0.1521	0.1022	0.0727	0.0715	0.0475	0.2	2038	0.1307	0.0891	0.0614	0.0631	0.0413
Relative IV QS	0.1000	0.0634	0.0975	0.0692	0.2626	0.1848	0.0	0890	0.0588	0.0983	0.0712	0.2980	0.2034
Hsieh & Jarrow IV QS	0.1105	0.0767	0.1115	0.0837	0.3401	0.2729	0.1	1144	0.0826	0.1124	0.0858	0.3366	0.2725
Absolute IV QS	0.0302	0.0276	0.0305	0.0316	0.0890	0.0822	0.0	0312	0.0304	0.0321	0.0385	0.1029	0.1239
QS rel. optionality	0.2200	0.1127	0.1276	0.0796	0.3164	0.0946	0.1	1883	0.1047	0.1276	0.0770	0.3322	0.0899
Chaudhury QS	0.1344	0.0891	0.1710	0.1198	0.2736	0.1826	0.1	1188	0.0811	0.1707	0.1197	0.2901	0.2005
Elasticity adj. QS	0.0118	0.0106	0.0069	0.0070	0.0076	0.0071	0.0	0261	0.0266	0.0351	0.0399	0.0631	0.0771
Amihud (hed. $+$ lev. adj.)	0.0009	0.0008	0.0004	0.0004	0.0003	0.0003	0.0	0008	0.0007	0.0005	0.0004	0.0003	0.0003
Pastor Stambaugh (hed. $+$ lev. adj.)	-0.0037	0.4561	-0.0003	0.1454	-0.0124	0.1212	0.0	0021	0.4739	0.0053	0.1662	-0.0057	0.1510

### Table 4: Correlations with possible drivers of option liquidity (ATM options).

The high-frequency measures are computed from LiveVol trade data on a monthly basis. All high-frequency measures are described in Section 3.1. As possible drivers of option liquidity we consider the quoted spread of the underlying, the implied volatility that corresponds to the mid price of the option, the level of the VIX index, the TED Spread, which is the difference between the 3-Month LIBOR and the 3-Month Treasury Bill, and the market capitalization of the underlying. Both, quoted spread of the underlying and mid IV are measured at time of the trade, where we use a binomial tree in the setting of Cox et al. (1979) to calculate the mid IV. The resulting intraday observations are aggregated to a monthly measure in the same manner as the high-frequency measures. VIX, TED Spread and the market capitalization of the underlying are measured at the end of the observation month. ATM call options have a delta in the range of  $0.375 < \Delta \leq 0.625$  and ATM put options a delta in the range of  $-0.625 < \Delta \leq -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Puts							
Panel A: Time-series co	orrelations							
	Underlying QS	Mid IV at trade	VIX	TED Spread	Underlying QS	Mid IV at trade	VIX	TED Spread
Quoted spread	0.5691	0.1453	0.1964	-0.0021	0.4022	-0.0394	0.0289	-0.1124
Relative IV QS	0.4490	-0.0051	0.0437	-0.1008	0.5183	0.1083	0.1739	0.0111
Hsieh & Jarrow IV QS	0.1016	-0.2709	-0.2748	-0.2755	0.2335	-0.1167	-0.0974	-0.1392
Absolute IV QS	0.9015	0.7804	0.7807	0.4390	0.9013	0.7531	0.7625	0.4571
QS rel. optionality	0.4367	0.0080	0.0544	-0.0670	0.4825	0.1130	0.1449	0.1122
Chaudhury QS	0.4449	-0.0147	0.0373	-0.0903	0.5113	0.0994	0.1715	0.0205
Elasticity adj. QS	0.9028	0.7996	0.8030	0.4653	0.9158	0.7773	0.7868	0.4696
$Underlying \ QS$	-	0.7493	0.7465	0.3412	-	0.7742	0.7718	0.3961

Panel B: Averag	e cross-sectional	correlations
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	Underlying QS	Mid IV at trade	Market capitalization	Underlying QS	Mid IV at trade	Market capitalization
Quoted spread	0.4105	-0.0939	-0.4321	0.3836	-0.0866	-0.3807
Relative IV QS	0.3986	-0.1248	-0.4125	0.4013	-0.0513	-0.3996
Hsieh & Jarrow IV QS $$	0.3741	-0.1083	-0.3913	0.3720	-0.0388	-0.3797
Absolute IV QS	0.5854	0.3400	-0.5644	0.5707	0.4084	-0.5533
QS rel. optionality	0.3942	-0.1344	-0.4188	0.4014	-0.0596	-0.4082
Chaudhury QS	0.4046	-0.1253	-0.4166	0.4069	-0.0566	-0.4007
Elasticity adj. QS	0.5897	0.3463	-0.5709	0.5790	0.4125	-0.5548
$Underlying \ QS$	-	0.3815	-0.5140	-	0.4025	-0.4891

# Table 5: Correlation between daily and monthly selected samples by moneyness category.

This table shows time-series and cross-sectional correlations between monthly high-frequency measures that are based on monthly and daily selected samples. The samples are selected based on the moneyness, which is defined by the absolute value of the delta: A call option is treated as OTM if  $0.125 < \Delta \leq 0.375$ , ATM if  $0.375 < \Delta \leq 0.625$ , or ITM if  $0.625 < \Delta \leq 0.875$ , whereas a put option is treated as ITM if  $-0.875 < \Delta \leq -0.625$ , ATM if  $-0.625 < \Delta \leq -0.375$ , or OTM if  $-0.375 < \Delta \leq -0.125$ . The daily selection takes place at the end of the previous trading day, the monthly selection at the end of the previous month. The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a column, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

		Calls			Puts	
	OTM	ATM	ITM	OTM	ATM	ITM
Panel A: Time-series co	orrelations					
Quoted spread	0.5702	0.6407	0.8864	0.6490	0.6503	0.9483
Relative IV QS	0.9004	0.6341	0.6388	0.8279	0.6838	0.6520
Hsieh & Jarrow IV QS	0.9297	0.6188	0.6181	0.8559	0.6559	0.6335
Absolute IV QS	0.9549	0.7201	0.5481	0.9248	0.8839	0.8616
QS rel. optionality	0.6406	0.6950	0.5556	0.7618	0.6849	0.6442
Chaudhury QS	$\left[ 0.9525 \right]$	0.8616	0.9251	0.8545	0.8492	0.9111
Elasticity adj. QS	0.9403	0.9672	0.9807	0.9426	0.9832	0.9834
Panel B: Average cross-	sectional co	prrelations				
Quoted spread	0.6722	0.6773	0.8119	0.7356	0.7247	0.8095
Relative IV QS	0.8734	0.6301	0.5576	0.8842	0.6060	0.5150
Hsieh & Jarrow IV QS	0.8794	0.6259	0.5389	0.8708	0.6634	0.5359
Absolute IV QS	0.8837	0.6636	0.5407	0.8746	0.7080	0.6181
QS rel. optionality	0.7319	0.5957	0.6073	0.7659	0.6147	0.5859
Chaudhury QS	0.9678	$\boxed{0.9092}$	0.8751	0.9594	0.8730	0.8287
Elasticity adj. QS	0.9125	0.9132	0.9194	0.9178	0.9043	0.9148

Table 6: Comparison of approximation methods for IV and delta (ATM options). The high-frequency measures and the corresponding high-frequency approximations are computed from LiveVol trade data on a monthly basis. All high-frequency measures are described in Section 2.1. For the high-frequency measures, the implied volatility and the corresponding delta are calculated using a binomial tree in the setting of Cox et al. (1979). The approximations for the implied volatility of the bid, ask, and mid price and the mid delta at the time of the trade are "BS div", for which the Black and Scholes (1973) model is employed and the present value of the dividend payment is subtracted from the underlying price, "BS std", for which the Black and Scholes (1973) model is employed without adjusting the underlying price for the dividend payment, and "Taylor", which uses a Taylor expansion to approximate the implied volatilities and delta at the time of the trade (using the delta, gamma and vega of the option at the end of the previous trading day). ATM call options have a delta in the range of  $0.375 < \Delta \le 0.625$  and ATM put options a delta in the range of  $-0.625 < \Delta \le -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

		Calls			Puts	
	BS div	BS std	Taylor	BS div	BS std	Taylor
Panel A: Time-series correlation						
Relative IV QS	0.9990	0.9991	0.9990	0.9992	0.9981	0.9971
Hsieh & Jarrow IV QS	0.9998	0.9999	0.9987	0.9997	0.9997	0.9969
Absolute IV QS	1.0000	1.0000	0.9997	1.0000	0.9999	0.9988
Chaudhury QS	0.9996	0.9989	0.9998	0.9998	0.9989	0.9999
Elasticity adj. QS	1.0000	1.0000	0.9998	0.9999	0.9999	1.0000
Panel B: Average cross-sectional con	rrelation					
Relative IV QS	0.9973	0.9974	0.9967	0.9999	0.9921	0.9916
Hsieh & Jarrow IV QS	0.9995	0.9997	0.9965	0.9999	0.9991	0.9918
Absolute IV QS	0.9997	0.9998	0.9971	0.9999	0.9993	0.9938
Chaudhury QS	0.9989	0.9968	0.9995	1.0000	0.9954	0.9993
Elasticity adj. QS	0.9997	0.9998	0.9994	0.9986	0.9989	0.9994
Panel C: Mean bias relative to samp	ole mean					
Relative IV QS	-0.0135	0.0192	-0.0198	-0.0059	-0.0341	-0.0283
Hsieh & Jarrow IV QS	-0.0047	-0.0028	-0.0458	-0.0027	-0.0063	-0.0509
Absolute IV QS	-0.0045	-0.0028	-0.0442	-0.0026	-0.0061	-0.0492
Chaudhury QS	-0.0091	0.0222	0.0252	-0.0031	-0.0284	0.0227
Elasticity adj. QS	-0.0092	-0.0080	-0.0004	0.0225	0.0207	0.0046
Panel D: RMSE relative to sample a	nean					
Relative IV QS	0.0630	0.0596	0.0717	0.0318	0.0997	0.1138
Hsieh & Jarrow IV QS	0.0371	0.0215	0.0989	0.0331	0.0472	0.1353
Absolute IV QS	0.0288	0.0154	0.0953	0.0274	0.0466	0.1611
Chaudhury QS	0.0383	0.0638	0.0455	0.0067	0.0734	0.0511
Elasticity adj. QS	0.0244	0.0188	0.0437	0.0441	0.0408	0.0425

# Table 7: Low-frequency proxies compared to high-frequency measures (ATM call options).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). ATM call options have a delta in the range of  $0.375 < \Delta \leq 0.625$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Analo	ogous to high-	frequency me	asure	Alt	ernative pro	xies
-	Exact	BS div	BS std	Taylor	Amihud	$_{\rm PS}$	QS
Panel A: Time-series correlation	on						
Quoted spread	0.9125	-	-	-	0.4999	0.2126	0.9125
Relative IV QS	0.9066	0.8968	0.9107	0.9056	0.3428	0.2061	0.8424
Hsieh & Jarrow IV QS	0.9112	0.9074	0.9092	0.9105	-0.003	0.1976	0.5762
Absolute IV QS	0.9865	0.9861	0. <b>9863</b>	0.9865	0.9059	0.1104	0.7862
QS rel. optionality	0.9091	-	-	-	0.3641	0.2134	0.8545
Chaudhury QS	0.8955	0.8901	0.9022	0.8955	0.3209	0.2060	0.8296
Elasticity adj. QS	0.9866	0.9865	0.9865	0.9866	0.9173	0.1088	0.7749
Panel B: Average cross-section	al correlation	ı					
Quoted spread	0.9545	-	-	-	0.4437	-0.0209	0.9545
Relative IV QS	0.9468	0.9443	0.9433	0.9471	0.4121	-0.0223	0.9252
Hsieh & Jarrow IV QS	0.9457	0.9457	0.9454	0.9464	0.3619	-0.0189	0.8712
Absolute IV QS	0.9535	$\begin{bmatrix} 0.9535 \end{bmatrix}$	0.9533	0.9538	0.6601	-0.0136	0.8046
QS rel. optionality	0.9556	-	-	-	0.4097	-0.0207	0.9301
Chaudhury QS	0.9542	0.9526	0.9509	0.9542	0.4005	-0.0221	0.9289
Elasticity adj. QS	0.9608	0.9605	0.9603	0.9608	0.6615	-0.0121	0.8185
Panel C: Mean bias relative to	sample mean	ı					
Quoted spread	0.2520	-	-	-	-	-	-
Relative IV QS	0.2784	0.3050	0.2561	0.2737	-	-	-
Hsieh & Jarrow IV QS	0.2814	0.2773	0.2723	0.2760	-	-	-
Absolute IV QS	0.3161	0.3127	0.3088	0.3105	-	-	-
QS rel. optionality	0.3134	-	-	-	-	-	-
Chaudhury QS	0.2654	0.2949	0.2527	0.2654	-	-	-
Elasticity adj. QS	0.2875	0.2792	0.2770	0.2874	-	-	-
Panel D: RMSE relative to san	nple mean						
Quoted spread	0.4220	-	-	-	-	-	-
Relative IV QS	0.4608	0.4407	0.4922	0.4549	-	-	-
Hsieh & Jarrow IV QS	0.4800	0.4707	0.4763	0.4716	-	-	-
Absolute IV QS	0.6623	0.6546	0.6581	0.6478	-	-	-
QS rel. optionality	0.4310	-	-	-	-	-	-
Chaudhury QS	0.4310	0.4208	0.4660	0.4310	-	-	-
Elasticity adj. QS	0.5874	0.5764	0.5805	0.5873	-	-	-

## A Filters

## A.1 Filters applied to the LiveVol data

Before merging the intraday trade data from LiveVol with OptionMetrics, we apply a minimal set of error filters to the LiveVol data. We filter out option trades for which the trade price or the number of contracts is negative, zero, or above 10 million. Furthermore, we delete entries on Saturdays, entries with multiple underlying symbols for the same root, duplicates, and cancelled trades.

After this merge, we apply a two-step filtering process as discussed in Section 1. The first step includes basic error filters that are presented in Panel A of Table 1:

*Outside trade hours:* We exclude all observations that take place before 9:30 ET and after 16:00 ET and therefore are outside the normal trading period (see, e.g., Andersen et al., 2021; Muravyev, 2016).

*Negative spread or zero bid:* We exclude all observations with a negative option quoted bid-ask spread (i.e., the ask price is lower than the bid price of the option), as well as all observations where the bid price of the option is negative (see, e.g., Andersen et al., 2021; Engle and Neri, 2010; Goyal and Saretto, 2009).

*Non-standard options:* We exclude all observations that belong to options that have a non-standard settlement or a non-standard expiration date (see, e.g., Frazzini and Pedersen, 2022).

Trade price severely outside quote range: We exclude all observations where the trade price is lower than the current bid price minus the current dollar bid-ask spread, or higher than the current ask price plus the current dollar bid-ask spread at the time of the trade, following Andersen et al. (2021).

*Huge deviation of bid and ask price:* We exclude all observations where the bid price is more than five times larger than the ask price, following Andersen et al. (2021),

*No-arbitrage relations violated:* Finally, we also exclude all observations, where the option prices violate arbitrage bounds in a tradeable way (compare, e.g., Goyal and Saretto, 2009; Goyenko and Zhang, 2021). Specifically, we remove a call observation, if the bid price of the call option is larger than the underlying ask price or the ask price of the option is smaller than the difference between the underlying bid price and the strike price. For put options, we exclude an observation, if the bid price of the put is larger than the strike price or the ask price of the put is smaller than the difference of the strike price and the bid price of the underlying.

The filters in the second step, presented in Panel B of Table 1, ensure that we can always calculate all high-frequency measures for every observation:

*Missing zero rate:* There are seven dates with option trades but missing zero rates from OptionMetrics. These dates are state dependent holidays. We exclude these observations form our analysis since we cannot calculate IVs without information on the risk-free rate.

Non-standard distribution or more than one dividend: We exclude all option series with nonstandard distributions (like stock splits or spin-offs) or more than one regular dividend payment of the underlying until expiration of the option to keep the complex calculations for determining the IVs as simple as possible.

*Filters related to underlying prices:* We exclude observations where the underlying ask price is lower than the bid price, the underlying bid price is zero, or the underlying bid price is more than five times the ask price. These conditions likely indicate erroneous underlying prices, making a meaningful calculation of the IV impossible.

*Filters related to mid IV:* We remove all observations where the option mid price violates noarbitrage bounds. Specifically, we remove an observation if, either the mid price of the options is lower than the intrinsic value or larger than the mid price of the underlying (the strike price) for calls (puts).<sup>18</sup> To account for an early exercise of the option, we use a more refined definition of the intrinsic value

$$IntrVal^{Call} = \max \{ S^{mid} - Div - K \exp(-r_f T); \quad S^{mid} - K \exp(-r_f t^{Div}); \quad 0 \}, \\ IntrVal^{Put} = \max \{ K \exp(-r_f t^{Div}) - S^{mid} + Div; \quad K - S^{mid}; \quad 0 \},$$
(15)

where  $S^{mid}$  denotes the mid price of the underlying, Div is the dollar dividend, K is the strike price of the option,  $r_f$  the risk-free rate, T the time to expiration of the option and  $t^{Div}$  the time until the dividend payment. The second term in the formula for calls and the first term in the formula for puts in Equation 15 represent the case of an early exercise, which is optimal for calls right before and for puts directly after the dividend payment. Additionally, we follow Engle and Neri (2010) and exclude all observations with  $0 \leq IV^{Mid} < 0.001$  or  $IV^{Mid} > 9$ , where  $IV^{Mid}$  denotes the IV of the option's mid price calculated via the binomial tree model.

Less than eight obs. per underlying: Finally, after applying all filters, we follow Schestag et al. (2016) and remove all observations from the high-frequency LiveVol sample and the corresponding low-frequency OptionMetrics sample for an underlying in a month if there are fewer than eight observations across all option series in either sample.<sup>19</sup>

<sup>&</sup>lt;sup>18</sup>In these cases, arbitrage opportunities cannot be exploited, unlike the real arbitrage opportunities of the first filtering step. Though, if any of these conditions are met, calculating an IV that matches the option's mid price is impossible.

<sup>&</sup>lt;sup>19</sup>Here, we examine all six moneyness samples separately. For example, if there are only three option trades in the LiveVol ATM call sample for a specific underlying in a specific month, we exclude all observations for this underlying in both the LiveVol and OptionMetrics ATM call samples for this month.

## A.2 Filters applied to the OptionMetrics data

Before using the data form OptionMetrics, we apply a similar set of filters as for the LiveVol data. Analogous to the LiveVol filters, we filter observations of *non-standard options*, observations with *negative spread or zero bid* or observations with a *huge deviation of bid and ask price*.

Due to the different data structure, the basic error filters differ slightly between the LiveVol data and the OptionMetrics data. First, we have to adjust the conditions for the no-arbitrage relations: Since OptionMetrics does not provide information on the bid and ask prices of the underlying, we have to use the closing underlying price for checking the no-arbitrage relations. Second, we also apply some further error filters to the OptionMetrics data: We exclude all observations with missing delta, following Frazzini and Pedersen (2022). We also exclude all observations with impossible quoted deltas (i.e., if the call delta is negative or above one and the put delta is positive or below minus one), and negative or missing contract size.

We apply the same second-step filters to our OptionMetrics samples as we do for the LiveVol samples, with the main difference being that there is no need for *filters related to underlying prices* since OptionMetrics does not provide bid and ask prices for the underlyings. Additionally, we add a *filter related to the mid IV*: A small number of quoted IVs in OptionMetrics (about 19,000 out of 23 million in our combined OptionMetrics samples) cannot be replicated with the quoted option prices and dividend information.<sup>20</sup> We exclude these observations from our analysis.

## **B** Details on the calculation of the implied volatilities

For reasons of comparability between the liquidity measures based on implied volatilities calculated from intraday and daily data, we aimed for keeping the calculation of the implied volatilites and deltas as close to the method of OptionMetrics as possible. Therefore, we use a binomial tree model in the setting of Cox et al. (1979) and define the necessary input parameters as follows: We assume that each year consists of 365 days. Options that expire before February 2015, usually have an expiration on the third Saturday of the month, even though trading typically ended the preceding Friday. Accounting for this change, we adjust the time to expiration and subtract one day, if the expiration date falls on a Saturday. Further, we linearly interpolate the provided zero curve of OptionMetrics to get the risk-free rate. For constructing the binomial tree, we use 500 time steps and employ the Newton-Raphson method to iteratively solve for implied volatilities. This parametrization successfully replicates the implied volatilities reported by OptionMetrics.

<sup>&</sup>lt;sup>20</sup>To our knowledge, this is due to inconsistencies in the dividend data provided by OptionMetrics.

In order to account for possible dividend payments of the underlying stock, we restrict our analysis to options written on underlyings that at most pay one regular dividend before the option's expiration. Following OptionMetrics, we model discrete dividend payments assuming a constant dividend yield when constructing the binomial tree. Furthermore, if the observation date coincides with the ex-dividend date of the underlying, we do not include this dividend in the binomial tree, presuming that the ex-dividend impact was already incorporated at the beginning of trading and thus is reflected in the quoted prices.

## C Additional results

### C.1 Low-frequency measures for OTM and ITM calls and ATM puts

Tables A1 and A2 complement Table 7 by adding results for low-frequency proxies for ITM and OTM call options. The main findings for ATM call options also hold for ITM and OTM call options: While the best proxy for the different high-frequency measures varies, all proxies that are analogous to their high-frequency counterparts exhibit high correlations that are similar in size. The Elasticity adj. QS and Absolute IV QS show the highest correlations in the time-series (Panel A). In the cross-sectional analysis (Panel B), the Chaudhury QS overall performs similar to these two measures. Regarding the alternative proxies, the Pástor and Stambaugh (2003) measure performs even worse OTM and ITM, exhibiting either insignificant or significantly negative correlations. The Amihud (2002) liquidity measure is the best alternative proxy for the Elasticity adj. QS and Absolute IV QS to capture time-series dynamics. The low-frequency version of the traditional quoted spread performs best for all other high-frequency measures and to assess cross-section variation. Looking at the approximation errors in Panels C and D, we find that the upward bias of using daily measures is more severe ITM and OTM compared to ATM.

For ATM put options, the results of the low-frequency approximation analysis are presented in Table A3. In essence, the qualitative results are the same as for ATM call options presented in Table 7. The main difference are slightly higher approximation errors of the proxies for put options.

### C.2 Effective-spread based measures

Effective spreads are often favored for their ability to reflect the actual costs faced by traders. Table A4 provides an overview of liquidity levels and standard deviations of the effective-spreadbased measures.

#### Suitable high-frequency measures

Table A5 compares correlations between effective-spread-based measures and variables commonly known to be related to liquidity. Analogous to the correlations of the quoted spread measures in Table 4, the time-series correlations with drivers of option liquidity in Panel A are mostly highest for the Elasticity adj. ES. The only exception are the correlations with the Underlying QS of call options, where the Absolute IV ES shows slightly higher correlations. As for the quoted-spread-based measures, the differences between the Elasticity adj. ES and Absolute IV ES are small. Compared to quoted-spread-based measures, effective-spread-based measures show lower correlations with the Underlying QS but higher correlations with all other economic variables (implied volatility at time of the trade, VIX, and TED Spread). A second notable difference is that all effective-spread-based measures, except the Hsieh & Jarrow ES, exhibit positive correlations with implied volatility at the time of trade, VIX, and TED Spread. However, these correlations are still relatively low compared to those of the Elasticity adj. ES and Absolute IV ES. In the cross-sectional analysis presented in Panel B, effectivespread-based measures produce results consistent with those of quoted-spread-based measures, with only minor differences in the magnitude of correlations.

Table A4 provides a comparison of the liquidity levels for the effective-spread-based measures. Essentially, the qualitative relations of liquidity levels across moneyness are almost identical to those for the quoted-spread-based measures in Table 3, with the only deviation that the Hsieh & Jarrow IV ES and the Absolute IV ES both identify ATM options as most liquid instead of OTM options, while both still have very similar liquidity levels. Notably, liquidity levels of all effective-spread-based measures are lower than their quoted-spread-based counterparts.

Table A6 compares correlations between daily and monthly samples for effective-spreadbased measures. As for the quoted-spread-based measures in Table 5, the Elasticity adj. ES is the only liquidity measure that exhibits consistently high correlations above 0.9. For the cross-sectional correlations in Panel B, the Elasticity adj. ES and the Chaudhury ES perform similarly, with both taking the win in three out of the six cases and correlations between 0.82 and 0.96.

Overall, the results for the effective-spread based measures are quantitatively and qualitatively very similar to those of the quoted-spread based measures with the Elasticity adj. ES clearly performing most consistently across the different analyses.

#### Approximation methods for IV and delta

Table A7 present the results comparing approximation methods for implied volatilities and deltas that are used to approximate our effective-spread-based high-frequency measures. The qualitative results of this analysis are essentially the same as those for the quoted-spread-based measures presented in Table 6.

#### Low-frequency measures

When it comes to calculating liquidity measures from low-frequency data, often the only possibility is to use quoted closing prices – trade prices (and therefore also effective-spread-based proxies) usually are not available. This makes it particularly interesting to evaluate the ability of our quoted-spread-based low-frequency proxies to approximate effective-spread-based highfrequency measures. Table A8 presents the results of this analysis for ATM call options.

Regarding the time-series correlations in Panel A, the only effective-spread-based high-frequency measures that can be approximated meaningfully by quoted-spread-based low-frequency proxies are the Elasticity adj. ES (correlations above 0.9) and Absolute IV ES (correlations slightly below 0.9). The correlations for all other high-frequency measures are much lower (less than 0.66). In the cross-section (Panel B) the quoted-spread-based proxies that are analogous to their high-frequency counterpart work much better (correlations above 0.9). Regarding the alternative proxies, the qualitative results are the same as in Table 7 for the quoted-spread-based high-frequency measures.

A significant difference is observed in the approximation errors for the effective-spreadbased high-frequency measures, as shown in Panels C and D of Table A8. These errors are substantially higher compared to those in Table 7. This can be attributed to the fact that effective spreads are, on average, lower than quoted spreads, which amplifies the upward bias that already exists for the quoted-spread-based low-frequency measures.

For ATM put options, the results presented in Table A9 are qualitatively the same as for ATM call options presented in Table A8. As for the quoted-spread based measures, the main difference to the results for call options are lightly higher approximation errors of the low-frequency proxies for put options.

To summarize, quoted-spread-based low-frequency approximations can even be used to approximate the Elasticity adj. ES and Absolute IV ES as long as only time-series dynamics and and cross-sectional variation are of interest. Regarding the absolute magnitude of the measures, the approximation leads to substantial biases.

# Table A1: Low-frequency proxies compared to high-frequency measures (ITM call options).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). ITM call options have a delta in the range of  $0.625 < \Delta \leq 0.875$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Analo	ogous to high-	frequency me	asure	Alt	ternative proz	cies
-	Exact	BS div	BS std	Taylor	Amihud	$_{\rm PS}$	QS
Panel A: Time-series correlation	on						
Quoted spread	0.9599	-	-	-	0.4939	-0.2506	0.9599
Relative IV QS	0.9565	0.9443	0.9604	0.9450	0.2183	-0.2606	0.8721
Hsieh & Jarrow IV QS	0.9499	0.9444	0.9520	0.9416	0.0371	-0.2249	0.7404
Absolute IV QS	0.9743	0.9732	0.9751	0.9693	0.8505	-0.1742	0.7861
QS rel. optionality	0.9381	-	-	-	-0.0443	-0.0546	0.6900
Chaudhury QS	0.9578	0.9549	0.9604	0.9578	0.2922	-0.2513	0.9033
Elasticity adj. QS	0.9868	0.9868	0.9849	0.9868	0.9155	-0.1560	0.7279
Panel B: Cross-sectional correl	lation						
Quoted spread	0.9148	-	-	-	0.4718	-0.0915	0.9148
Relative IV QS	0.8240	0.7699	0.8066	0.7730	0.3579	-0.0850	0.7837
Hsieh & Jarrow IV QS	0.8292	0.8035	0.8231	0.8052	0.3100	-0.0729	0.7345
Absolute IV QS	0.8447	0.8305	0.8400	0.8242	0.6247	-0.0770	0.7130
QS rel. optionality	0.8986	-	-	-	0.2608	-0.0771	0.7237
Chaudhury QS	0.8896	0.8819	0.8823	0.8896	0.4077	-0.0914	0.8766
Elasticity adj. QS	0.9156	0.9149	0.8952	0.9156	0.6908	-0.0777	0.7615
Panel C: Mean bias relative to	sample mean	ı					
Quoted spread	0.3039	-	-	-	-	-	-
Relative IV QS	0.6637	0.5085	0.7290	0.6110	-	-	-
Hsieh & Jarrow IV QS	0.7590	0.6383	0.7540	0.6628	-	-	-
Absolute IV QS	0.7850	0.6800	0.7812	0.6873	-	-	-
QS rel. optionality	0.3630	-	-	-	-	-	-
Chaudhury QS	0.4570	0.4182	0.5035	0.4570	-	-	-
Elasticity adj. QS	0.4580	0.4659	0.4783	0.4581	-	-	-
Panel D: RMSE relative to sar	nple mean						
Quoted spread	0.5098	-	-	-	-	-	-
Relative IV QS	0.9688	0.8616	1.0391	1.0836	-	-	-
Hsieh & Jarrow IV QS	1.1853	1.0832	1.1695	1.1667	-	-	-
Absolute IV QS	1.2916	1.2036	1.2778	1.2286	-	-	-
QS rel. optionality	0.4044	-	-	-	-	-	-
Chaudhury QS	0.6958	0.6592	0.7497	0.6959	-	-	-
Elasticity adj. QS	0.8334	0.8411	0.8714	0.8336	-	-	-

# Table A2: Low-frequency proxies compared to high-frequency measures (OTM call options).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). OTM call options have a delta in the range of  $0.125 < \Delta \leq 0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Analo	ogous to high-	frequency me	asure	Alt	ernative pro	xies
	Exact	BS div	BS std	Taylor	Amihud	$_{\rm PS}$	QS
Panel A: Time-series correle	ation						
Quoted spread	0.9353	-	-	-	0.3025	-0.0016	0.9353
Relative IV QS	0.9204	0.9187	0.9231	0.9199	0.2359	-0.0033	0.8776
Hsieh & Jarrow IV QS	0.9281	0.9285	0.9266	0.9285	-0.1801	0.1255	0.5754
Absolute IV QS	0.9930	0.9931	0.9929	0.9930	0.8934	-0.1636	0.6700
QS rel. optionality	0.9349				0.2598	0.0151	0.9264
Chaudhury QS	0.8973	0.8950	0.9028	0.8973	0.2257	-0.0136	0.8306
Elasticity adj. QS	0.9930	0.9926	0.9929	0.9930	0.8938	-0.1530	0.6632
Panel B: Cross-sectional cor	relation						
Quoted spread	0.9354	-	-	-	0.4286	-0.0120	0.9354
Relative IV QS	0.9531	0.9526	0.9530	0.9533	0.4085	-0.0133	0.9313
Hsieh & Jarrow IV QS	0.9501	0.9501	0.9493	0.9504	0.3709	-0.0077	0.8691
Absolute IV QS	0.9606	0.9605	0.9602	0.9608	0.6824	-0.0089	0.7979
QS rel. optionality	0.9501	-	-	-	0.4155	-0.0130	0.9394
Chaudhury QS	0.9598	0.9592	0.9585	0.9598	0.3863	-0.0141	0.9045
Elasticity adj. QS	0.9553	0.9526	0.9549	0.9553	0.6842	-0.0078	0.7995
Panel C: Mean bias relative	to sample mean	ı					
Quoted spread	0.3871	-	-	-	-	-	-
Relative IV QS	0.2986	0.2951	0.3102	0.2892	-	-	-
Hsieh & Jarrow IV QS	0.3000	0.3021	0.2917	0.2905	-	-	-
Absolute IV QS	0.3266	0.3284	0.3196	0.3165	-	-	-
QS rel. optionality	0.3241	-	-	-	-	-	-
Chaudhury QS	0.2397	0.2342	0.2602	0.2397	-	-	-
Elasticity adj. QS	0.3822	0.3530	0.3731	0.3816	-	-	-
Panel D: RMSE relative to a	sample mean						
Quoted spread	0.5206	-	-	-	-	-	-
Relative IV QS	0.4250	0.4373	0.4221	0.4120	-	-	-
Hsieh & Jarrow IV QS	0.4532	0.4459	0.4553	0.4398	-	-	-
Absolute IV QS	0.5458	0.5394	0.5477	0.5285	-	-	-
QS rel. optionality	0.3997	-	-	-	-	-	-
Chaudhury QS	0.3744	0.3961	0.3702	0.3744	-	-	-
Elasticity adj. QS	0.6082	0.5997	0.5827	0.6075	-	-	-

# Table A3: Low-frequency proxies compared to high-frequency measures (ATM put options).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). ATM put options have a delta in the range of  $-0.625 < \Delta \leq -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level.

		Analogou	us to BM		Alternative proxies		
	Exact	BS div	BS std	Taylor	Amihud	$_{\rm PS}$	QS
Panel A: Time-series correlatio	n						
Quoted spread	0.8961	-	-	-	0.3851	0.2155	0.8961
Relative IV QS	0.9343	0.9378	0.9285	0.9352	0.5470	0.2784	0.9142
Hsieh & Jarrow IV QS	0.9280	0.9298	0.9289	0.9295	0.2609	0.1941	0.7596
Absolute IV QS	0.9904	0.9908	0.9906	0.9912	0.9502	0.3097	0.6560
QS rel. optionality	0.9284	-	-	-	0.5128	0.2628	0.9262
Chaudhury QS	0.9192	0.9214	0.9125	0.9192	0.5269	0.2651	0.9039
Elasticity adj. QS	0.9863	0.9863	0.9863	0.9863	0.9452	0.2854	0.6469
Panel B: Cross-sectional correla	ntion						
Quoted spread	0.9457	-	-	-	0.3880	0.0234	0.9457
Relative IV QS	0.9278	0.9280	0.9215	0.9269	0.4145	0.0232	0.9033
Hsieh & Jarrow IV QS	0.9302	0.9300	$\begin{bmatrix} 0.9301 \end{bmatrix}$	$\begin{bmatrix} 0.9299 \end{bmatrix}$	0.3670	0.0241	0.8535
Absolute IV QS	0.9457	0.9455	0.9455	0.9452	0.6664	0.0187	0.7447
QS rel. optionality	0.9405	-	-	-	0.4026	0.0240	0.9023
Chaudhury QS	0.9393	0.9394	0.9337	0.9393	0.3958	0.0232	0.9153
Elasticity adj. QS	0.9563	0.9544	0.9548	0.9563	0.6610	0.0194	0.7698
Panel C: Mean bias relative to a	sample mean						
Quoted spread	0.2715	-	-	-	-	-	-
Relative IV QS	0.3399	0.3286	0.2843	0.3306	-	-	-
Hsieh & Jarrow IV QS	0.3388	0.3321	0.3237	0.3284	-	-	-
Absolute IV QS	0.3750	0.3692	0.3602	0.3636	-	-	-
QS rel. optionality	0.3741	-	-	-	-	-	-
Chaudhury QS	0.3224	0.3177	0.2818	0.3222	-	-	-
Elasticity adj. QS	0.3199	0.3434	0.3431	0.3198	-	-	-
Panel D: RMSE relative to sam	ple mean						
Quoted spread	0.4438	-	-	-	-	-	-
Relative IV QS	0.5623	0.5493	0.5126	0.5534	-	-	-
Hsieh & Jarrow IV QS	0.5789	0.5708	0.5616	0.5644	-	-	-
Absolute IV QS	0.8476	0.8395	0.8224	0.8183	-	-	-
QS rel. optionality	0.5004	-	-	-	-	-	-
Chaudhury QS	0.5149	0.5102	0.4814	0.5148	-	-	-
Elasticity adj. QS	0.6855	0.7066	0.7101	0.6855	-	-	-

# Table A4: Descriptive statistics for the high-frequency measures (effective-spread-based).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. All measures are calculated on a monthly basis. Moneyness is defined by the absolute value of the delta at the end of the previous trading day: A call option is treated as OTM if  $0.125 < \Delta \le 0.375$ , ATM if  $0.375 < \Delta \le 0.625$ , or ITM if  $0.625 < \Delta \le 0.875$ , whereas a put option is treated as ITM if  $-0.875 < \Delta \le -0.625$ , ATM if  $-0.625 < \Delta \le -0.375$ , or OTM if  $-0.375 < \Delta \le -0.125$ . The observation period is from January 1, 2004 to June 30, 2021.

	OTI	M	ATM		ITM	
_	Mean	SD	Mean	SD	Mean	SD
Panel A: Call options						
Effective Spread	0.1192	0.0792	0.0530	0.0353	0.0334	0.0193
Relative IV ES	0.0507	0.0331	0.0491	0.0326	0.0905	0.0598
Hsieh & Jarrow IV ES	0.0560	0.0404	0.0559	0.0397	0.1110	0.0786
Absolute IV ES	0.0153	0.0149	0.0152	0.0141	0.0292	0.0244
ES rel. optionality	0.1139	0.0695	0.0638	0.0408	0.1588	0.0743
Chaudhury ES	0.0713	0.0472	0.0871	0.0572	0.1137	0.0665
Elasticity adj. ES	0.0058	0.0056	0.0035	0.0033	0.0032	0.0027
Panel B: Put options						
Effective Spread	0.0928	0.0613	0.0440	0.0279	0.0283	0.0162
Relative IV ES	0.0431	0.0282	0.0459	0.0299	0.0914	0.0634
Hsieh & Jarrow IV ES	0.0546	0.0394	0.0525	0.0367	0.0986	0.0738
Absolute IV ES	0.0149	0.0144	0.0149	0.0154	0.0301	0.0346
ES rel. optionality	0.0906	0.0565	0.0597	0.0375	0.1708	0.0798
Chaudhury ES	0.0602	0.0401	0.0810	0.0515	0.1115	0.0652
Elasticity adj. ES	0.0056	0.0053	0.0035	0.0034	0.0032	0.0034

#### Table A5: Correlations with possible drivers of option liquidity (ATM options; effective-spread-based).

The high-frequency measures are computed from LiveVol trade data on a monthly basis. All high-frequency measures are described in Section 3.1. As possible drivers of option liquidity we consider the quoted spread of the underlying, the implied volatility that corresponds to the mid price of the option, the level of the VIX index, the TED Spread, which is the difference between the 3-Month LIBOR and the 3-Month Treasury Bill, and the market capitalization of the underlying. Both, quoted spread of the underlying and mid IV are measured at time of the trade, where we use a binomial tree in the setting of Cox et al. (1979) to calculate the mid IV. The resulting intraday observations are aggregated to a monthly measure in the same manner as the high-frequency measures. VIX, TED Spread and the market capitalization of the underlying are measured at the end of the observation month. ATM call options have a delta in the range of  $0.375 < \Delta \le 0.625$  and ATM put options a delta in the range of  $-0.625 < \Delta \le -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Calls				Puts				
Panel A: Time-series co	orrelations								
	Underlying QS	Mid IV at trade	VIX	TED Spread	Underlying QS	Mid IV at trade	VIX	TED Spread	
Effective spread	0.3623	0.3161	0.2305	0.3232	0.3144	0.1676	0.0813	0.2143	
Relative IV ES	0.2485	0.1730	0.0820	0.2293	0.4200	0.3133	0.2283	0.3431	
Hsieh & Jarrow IV ES	-0.0401	-0.0969	-0.2185	0.0147	0.1274	0.0572	-0.0628	0.1453	
Absolute IV ES	0.7800	0.8865	0.8195	0.6474	0.8280	0.8797	0.8192	0.6684	
ES rel. optionality	0.2513	0.2038	0.1121	0.2611	0.3474	0.3012	0.1916	0.4017	
Chaudhury ES	0.2348	0.1679	0.0816	0.2444	0.4110	0.3155	0.2371	0.3623	
Elasticity adj. ES	0.7697	0.8904	0.8289	0.6638	0.8309	0.8931	0.8329	0.6755	
Panel B: Average cross-	sectional correlatio	ns							
	Underlying Q	S Mid IV	at trade	Market capitalization	Underlying	QS Mid I	V at trade	Market capitalization	
Effective spread	0.4013	-0.0	815	-0.4217	0.3683	-0	.0776	-0.3734	
Relative IV QS	0.3889	-0.1	133	-0.4029	0.3851	-0	.0422	-0.3896	
Hsieh & Jarrow IV ES	0.3657	-0.0	952	-0.3828	0.3562	-0	.0309	-0.3703	
Absolute IV ES	0.5804	0.3	639	-0.5580	0.5598	0	.4256	-0.5480	
ES rel. optionality	0.3826	-0.1	217	-0.4035	0.3838	-0	.0486	-0.3944	
Chaudhury ES	0.3950	-0.1	143	-0.4066	0.3917	-0	0.0485	-0.3917	
Elasticity adj. ES	0.5852	0.3	706	-0.5648	0.5695	] [0	.4320	-0.5531	

# Table A6: Correlation between daily and monthly selected samples by moneyness category (effective-spread-based).

This table shows time-series and cross-sectional correlations between monthly high-frequency measures that are based on monthly and daily selected samples. The samples are selected based on the moneyness, which is defined by the absolute value of the delta: A call option is treated as OTM if  $0.125 < \Delta \leq 0.375$ , ATM if  $0.375 < \Delta \leq 0.625$ , or ITM if  $0.625 < \Delta \leq 0.875$ , whereas a put option is treated as ITM if  $-0.875 < \Delta \leq -0.625$ , ATM if  $-0.625 < \Delta \leq -0.375$ , or OTM if  $-0.375 < \Delta \leq -0.125$ . The daily selection takes place at the end of the previous trading day, the monthly selection at the end of the previous month. The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a column, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Calls				Puts	
	OTM	ATM	ITM	OTM	ATM	ITM
Panel A: Time-series co	orrelations					
Effective spread	0.7335	0.6364	0.7721	0.7911	0.6612	0.8285
Relative IV ES	0.9482	0.6866	0.4858	$\boxed{0.9313}$	0.7279	0.6605
Hsieh & Jarrow IV ES	0.9548	0.6688	0.4880	$\left[ 0.\overline{9322}  ight]$	0.7427	0.6703
Absolute IV ES	$0.\overline{9728}$	0.8369	0.4547	0.9427	0.8827	0.8795
ES rel. optionality	0.7939	0.7418	0.6011	0.8585	0.7714	0.7768
Chaudhury ES	0.9771	0.8894	0.8879	0.9207	0.8877	0.8908
Elasticity adj. ES	0.9637	0.9657	0.9832	$\boxed{0.9391}$	0.9701	0.9852
Panel B: Average cross-	sectional c	orrelations				
Effective spread	0.6399	0.6310	0.7611	0.6985	0.6888	0.7677
Relative IV ES	0.8688	0.6628	0.5454	0.8782	0.6505	0.4739
Hsieh & Jarrow IV ES	0.8696	0.6578	0.5153	0.8537	0.6968	0.4916
Absolute IV ES	0.8818	0.7022	0.5364	0.8696	0.7466	0.5733
ES rel. optionality	0.6877	0.5485	0.5771	0.7184	0.5664	0.5384
Chaudhury ES	0.9619	0.9114	0.8672	0.9503	0.8759	0.8179
Elasticity adj. ES	0.9058	0.8988	0.9065	0.9105	0.8954	0.8984

# Table A7: Comparison of approximation methods for IV and delta (ATM options; effective-spread-based).

The high-frequency measures and the corresponding high-frequency approximations are computed from LiveVol trade data on a monthly basis. All high-frequency measures are described in Section 2.1. For the high-frequency measures, the implied volatility and the corresponding delta are calculated using a binomial tree in the setting of Cox et al. (1979). The approximations for the implied volatility of the bid, ask, and mid price and the mid delta at the time of the trade are "BS div", for which the Black and Scholes (1973) model is employed and the present value of the dividend payment is subtracted from the underlying price, "BS std", for which the Black and Scholes (1973) model is employed without adjusting the underlying price for the dividend payment, and "Taylor", which uses a Taylor expansion to approximate the implied volatilities and delta at the time of the trade (using the delta, gamma and vega of the option at the end of the previous trading day). ATM call options have a delta in the range of  $0.375 < \Delta \le 0.625$  and ATM put options a delta in the range of  $-0.625 < \Delta \le -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

		Calls			Puts	
	BS div	BS std	Taylor	BS div	BS std	Taylor
Panel A: Timeseries correlation						
Relative IV ES	0.9994	0.9993	0.9992	0.9985	0.9975	0.9974
Hsieh & Jarrow IV ES	0.9999	1.0000	0.9990	0.9994	$\begin{bmatrix} 0.9993 \end{bmatrix}$	0.9975
Absolute IV ES	1.0000	1.0000	0.9997	0.9999	$\begin{bmatrix} 0.9999 \end{bmatrix}$	0.9993
Chaudhury ES	0.9998	0.9993	0.9999	0.9997	0.9990	0.9999
Elasticity adj. ES	1.0000	1.0000	0.9998	0.9999	0.9999	1.0000
Panal P. Cross spatianal correlate	i o m					
Pulatice IV EC			0.0005		0.0010	0.0004
	0.9974	0.9973	0.9965	0.9998	0.9910	0.9904
Hsien & Jarrow IV ES	0.9996	0.9998	0.9962	0.9999	0.9990	0.9897
Absolute IV ES	0.9997	0.9998	0.9965	0.9999	0.9993	0.9927
Chaudhury ES	0.9988	0.9967	0.9993	1.0000	0.9950	0.9989
Elasticity adj. ES	0.9997	0.9998	0.9991	0.9986	0.9989	0.9992
Panel C: Mean bias relative to sa	mple mean					
Relative IV ES	-0.0130	0.0191	-0.0190	-0.0058	-0.0334	-0.0262
Hsieh & Jarrow IV ES	-0.0041	-0.0027	-0.0451	-0.0027	-0.0059	-0.0489
Absolute IV ES	-0.0041	-0.0027	-0.0437	-0.0025	-0.0055	-0.0472
Chaudhury ES	-0.0091	0.0219	0.0251	-0.0031	-0.0283	0.0224
Elasticity adj. ES	-0.0092	-0.0079	-0.0003	0.0225	0.0207	0.0046
Panel D: RMSE relative to sample	le mean					
Relative IV ES	0.0596	0.0626	0.0729	0.0517	0.1057	0.1176
Hsieh & Jarrow IV ES	0.0357	0.0236	0.1012	0.0608	0.0731	0.1506
Absolute IV ES	0.0295	0.0179	0.1008	0.0434	0.0574	0.1582
Chaudhury ES	0.0383	0.0630	0.0467	0.0082	0.0733	0.0503
Elasticity adj. ES	0.0250	0.0181	0.0523	0.0439	0.0413	0.0470

# Table A8: Low-frequency proxies compared to high-frequency measures (ATM call options, effective-spread-based high-frequency measures).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). ATM call options have a delta in the range of  $0.375 < \Delta \leq 0.625$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level. Solid boxes give the best value in a row, and dashed boxes give numbers that are not significantly different from this value at the 5% level.

	Analo	ogous to high	-frequency me	asure	Alternative proxies		
	QS analog	BS div	BS std	Taylor	Amihud	PS	QS
Panel A: Time-series correla	ation						
Effective spread	0.6534	-	-	-	0.3789	-0.0168	0.6534
Relative IV ES	0.5727	0.5861	0.5702	0.5752	0.2329	-0.0303	0.5986
Hsieh & Jarrow IV ES	0.5880	0.5942	0.5913	0.5922	-0.0717	-0.0096	0.3854
Absolute IV ES	0.8889	0.8903	0.8891	0.8912	0.8449	0.0105	0.6143
ES rel. optionality	0.5835	-	-	-	0.2607	-0.0273	0.5861
Chaudhury ES	0.5984	0.6055	0.5920	0.5984	0.2087	-0.0431	0.5585
Elasticity adj. ES	0.9086	0.9088	0.9090	0.9086	0.8426	0.0094	0.5929
Panel B: Average cross-secti	onal correlation	l					
Effective spread	0.9291	-	-	-	0.4536	-0.0204	0.9291
Relative IV ES	0.9218	0.9191	0.9176	0.9219	0.4209	-0.0228	0.9010
Hsieh & Jarrow IV ES	0.9214	0.9214	0.9208	0.9220	0.3693	-0.0195	0.8463
Absolute IV ES	0.9309	0.9309	0.9306	0.9310	0.6698	-0.0136	0.7688
ES rel. optionality	0.9262	-	-	-	0.4249	-0.0215	0.9049
Chaudhury ES	0.9286	0.9268	0.9252	0.9286	0.4087	-0.0227	0.9044
Elasticity adj. ES	0.9384	0.9382	0.9381	0.9384	0.6720	-0.0120	0.7839
Panel C: Mean bias relative	to sample mean	ı					
Effective spread	0.9304	-	-	-	-	-	-
Relative IV ES	0.9852	0.9505	1.0264	0.9779	-	-	-
Hsieh & Jarrow IV ES	0.9940	0.9798	0.9877	0.9857	-	-	-
Absolute IV ES	1.0069	0.9957	1.0018	0.9983	-	-	-
ES rel. optionality	1.0010	-	-	-	-	-	-
Chaudhury ES	0.9627	0.9430	1.0085	0.9627	-	-	-
Elasticity adj. ES	0.9555	0.9396	0.9429	0.9553	-	-	-
Panel D: RMSE relative to s	ample mean						
Effective spread	1.2602	-	-	-	-	-	-
Relative IV ES	1.3365	1.2925	1.3907	1.3249	-	-	-
Hsieh & Jarrow IV ES	1.3812	1.3622	1.3729	1.3664	-	-	-
Absolute IV ES	1.6567	1.6423	1.6492	1.6331	-	-	-
ES rel. optionality	1.2537	-	-	-	-	-	-
Chaudhury ES	1.2925	1.2698	1.3528	1.2925	-	-	-
Elasticity adj. ES	1.5286	1.5090	1.5151	1.5284	-	-	-

# Table A9: Low-frequency proxies compared to high-frequency measures (ATM put options, effective-spread-based high-frequency measures).

The high-frequency measures are computed from LiveVol trade data and are described in Section 2.1. The low-frequency proxies are computed from OptionMetrics quote data and are described in Section 3.2. All measures are calculated on a monthly basis. The first four proxies are low-frequency versions of the high-frequency measures and they differ in how the implied volatility of the bid, ask, and mid price and the corresponding delta are calculated: The calculation is based on a binomial tree if appropriate ("Exact"), volatilities and delta are approximated with the Black and Scholes (1973) model, subtracting the present value of the dividend payment from the underlying price ("BS div"), Black and Scholes (1973) is employed without subtracting the value of the dividend payment from the underlying price ("BS std"), and a Taylor expansion is used ("Taylor"). The alternative low-frequency proxies are the Amihud (2002) liquidity measure ("Amihud"), the Pástor and Stambaugh (2003) measure ("PS"), and a low-frequency version of the relative quoted spread ("QS"). ATM put options have a delta in the range of  $-0.625 < \Delta \leq -0.375$  at the end of the previous trading day. The observation period is from January 1, 2004 to June 30, 2021. Bold numbers are statistically significant at the 5% level.

	Analogous to BM				Alternative proxies			
	QS analog	BS div	BS std	Taylor	Amihud	$_{\rm PS}$	QS	
Panel A: Time-series correlati	ion							
Effective spread	0.6986	-	-	-	0.2815	0.0225	0.6986	
Relative IV ES	0.6761	0.6630	0.6768	0.6765	0.4304	0.0808	0.6920	
Hsieh & Jarrow IV ES	0.6486	0.6399	0.6452	0.6511	0.1294	-0.0025	0.5259	
Absolute IV ES	0.9028	0.9017	0.9022	0.9063	0.8784	0.2051	0.4725	
ES rel. optionality	0.6994	-	-	-	0.3601	0.0597	0.5890	
Chaudhury ES	0.6944	0.6873	0.6977	0.6944	0.4111	0.0639	0.6562	
Elasticity adj. ES	0.9212	0.9212	0.9213	0.9212	0.8659	0.1828	0.4547	
Panel B: Cross-sectional corre	lation							
Effective spread	0.9127	-	-	-	0.3942	0.0217	0.9127	
Relative IV ES	0.8914	0.8909	0.8841	0.8901	0.4177	0.0193	0.8643	
Hsieh & Jarrow IV ES	0.8961	0.8951	0.8948	0.8952	0.3683	0.0205	0.8145	
Absolute IV ES	0.9180	0.9174	0.9172	0.9173	0.6722	0.0155	0.7001	
ES rel. optionality	0.9003	-	-	-	0.4138	0.0216	0.8602	
Chaudhury ES	0.9043	0.9044	0.8987	0.9043	0.3990	0.0208	0.8785	
Elasticity adj. ES	0.9300	0.9281	0.9285	0.9300	0.6687	0.0174	0.7282	
Panel C: Mean bias relative to	sample mean	ı						
Effective spread	1.0245	-	-	-	-	-	-	
Relative IV ES	1.1399	1.0512	1.1219	1.1252	-	-	-	
Hsieh & Jarrow IV ES	1.1398	1.1157	1.1291	1.1232	-	-	-	
Absolute IV ES	1.1503	1.1271	1.1412	1.1324	-	-	-	
ES rel. optionality	1.1369	-	-	-	-	-	-	
Chaudhury ES	1.1079	1.0432	1.1005	1.1077	-	-	-	
Elasticity adj. ES	1.0593	1.0955	1.0960	1.0592	-	-	-	
Panel D: RMSE relative to sat	mple mean							
Effective spread	1.3663	-	-	-	-	-	-	
Relative IV ES	1.5652	1.5448	1.4628	1.5454	-	-	-	
Hsieh & Jarrow IV ES	1.6006	1.5878	1.5705	1.5739	-	-	-	
Absolute IV ES	2.0743	2.0622	2.0338	2.0228	-	-	-	
ES rel. optionality	1.4014	-	-	-	-	-	-	
Chaudhury ES	1.4885	1.4810	1.4167	1.4883	-	-	-	
Elasticity adj. ES	1.7773	1.8146	1.8196	1.7772	-	-	-	